Coherence Dictionary for the Taiji–Skyrme Framework

Quantum Operator, Classical Order Parameter, and Taiji Spinor Mapping

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Abstract

The Taiji–Skyrme framework employs several related notions of coherence across its quantum, classical, and phenomenological formulations. At the quantum level, coherence is defined as a renormalised composite operator constructed from Skyrme currents. At the classical level, a scalar order parameter $\psi(x)$ encodes the effective degree of ordered structure imprinted on the Potential Field (Ψ). In the spinor representation, a Taiji spinor $Z(x) \in \mathbb{C}^2$ provides a convenient parametrisation in which the norm $|Z(x)|^2$ is proportional to $\psi(x)$. Finally, in coarse-grained regimes, a calibrated relation links coherence to the local energy density and hence to the gravitational sector via the stress–energy tensor.

The purpose of this document is to unify these notions into a single hierarchy of definitions. The quantum coherence operator $\hat{C}_R(x)$ is taken as primary. The classical coherence scalar is defined as its expectation value in Excitation-selected states, and the dimensionless coherence field $\psi(x)$ is obtained by normalisation with respect to a fixed reference scale. The Taiji spinor representation and energy-density-based normalisation are then treated as derived structures. This dictionary provides the canonical mapping used throughout the Taiji–Skyrme document suite and removes any ambiguity in the use of the term "coherence".

1 Introduction and Scope

The Taiji–Skyrme theory is formulated in terms of an $SU(2)$-valued field $U(x)$ on spacetime and an associated current algebra built from

$$L_\mu(x) = U^\dagger(x)\,\partial_\mu U(x). \tag{1}$$

In the quantum theory, these fields are promoted to operators and obey canonical commutation relations. The framework is designed to encode the action of the Excitation (Φ) on the Potential Field (Ψ), with coherence representing the extent to which Excitation-imposed order is locally realised in the field configuration.

Historically, three related but distinct objects have been used as "coherence":

- a renormalised composite operator constructed from the currents L_μ,
- the norm of a Taiji spinor $Z(x)$ that parametrises the $SU(2)$ field,
- and a scalar field $\psi(x)$ whose value is used in effective potentials, density hierarchies, and phenomenology.

In addition, certain documents employ a proportionality between coherence and energy density to calibrate normalisation constants.

The goal of this document is to impose a single, consistent chain of definitions:

$$U, L_\mu \;\longrightarrow\; \hat{C}_R(x) \;\longrightarrow\; C_{\rm cl}(x) \;\longrightarrow\; \psi(x) \;\longrightarrow\; C_{\rm phys}(x), \rho_{\rm coh}(x), \tag{2}$$

with the Taiji spinor $Z(x)$ entering as a parametrisation of $\psi(x)$ rather than as an independent definition. This chain is then used as a reference for all other Taiji–Skyrme documents.

2 Quantum Coherence Operator

2.1 Definition as a composite operator

The fundamental object that encodes local coherence at the quantum level is a renormalised composite operator $\hat{C}_R(x)$ constructed from the Skyrme currents. Its precise form depends on the chosen regularisation scheme, but schematically one has

$$\hat{C}_R(x) = \lim_{y\to x}\Big[\mathcal{F}\big(L_\mu(x), L_\nu(y)\big) - \mathcal{F}_{\rm sing}(x,y)\Big], \tag{3}$$

where:

- $\mathcal{F}(L_\mu(x), L_\nu(y))$ is a local functional of the currents that measures their quadratic or higher-order correlations,
- $\mathcal{F}_{\rm sing}(x,y)$ subtracts short-distance singularities and depends on the renormalisation prescription,
- the limit $y \to x$ is taken after subtraction, yielding a finite local operator.

This definition is compatible with the canonical commutation relations of the Skyrme field and is interpreted as a structural measure of how strongly ordered the local configuration is, in the sense of Excitation-imposed patterns.

2.2 Algebra and dimensions

The operator $\hat{C}_R(x)$ is hermitian and transforms trivially under global $SU(2)$ rotations, being constructed from traces of current bilinears. Its dimension is fixed by the dimension of the underlying Skyrme Lagrangian. For a four-dimensional theory, one typically has

$$[\hat{C}_R] = [\text{mass}]^4, \tag{4}$$

so that $\hat{C}_R$ has the same engineering dimension as an energy density.

The commutator

$$\big[\hat{C}_R(x), \hat{C}_R(y)\big] \tag{5}$$

defines a coherence kernel that encodes uncertainty relations and quantum fluctuations in coherence. In the present document, the exact form of this kernel is not required; it suffices that it is determined by the underlying current algebra and the chosen renormalisation scheme. The quantum operator $\hat{C}_R(x)$ is taken as the primary definition of coherence in the Taiji–Skyrme framework.

3 Classical Coherence Field and Order Parameter

3.1 Expectation value as classical coherence

Given a quantum state $|\Psi\rangle$ describing a configuration of the Taiji–Skyrme field, the classical coherence field is defined as the expectation value

$$C_{\text{cl}}(x) = \langle\Psi|\hat{C}_R(x)|\Psi\rangle. \tag{6}$$

In the philosophical interpretation of the theory, the state $|\Psi\rangle$ encodes the action of the Excitation (Φ) as an ordering principle acting on the Potential Field (Ψ). The field $C_{\text{cl}}(x)$ then represents the degree of ordered structure realised at the spacetime point x.

3.2 Dimensionless coherence order parameter

To use coherence as an order parameter in effective Lagrangians and phenomenology, it is convenient to introduce a fixed reference scale $C_\star$ with the same dimension as C_{cl} and define the dimensionless coherence field

$$\psi(x) = \frac{C_{\text{cl}}(x)}{C_\star}. \tag{7}$$

The choice of $C_\star$ sets the unit in which coherence is measured. In practice, $C_\star$ can be chosen so that $\psi(x)$ takes values of order unity in regimes of interest (e.g. in a reference vacuum branch or in a benchmark astrophysical environment).

The field $\psi(x)$ is the classical coherence order parameter used throughout the effective Taiji–Skyrme Lagrangians, domain-wall solutions, galaxy phenomenology, and cosmological models. Whenever the symbol "ψ" appears in the document suite, it refers to the dimensionless field defined by (7).

3.3 Coherence branches and branch–resolved quantities

In regimes where the effective potential $V_{\rm eff}(\psi)$ admits several local minima, we label the corresponding coherence branches by

$$\mathcal{B} = \{1, 2, \ldots, N\}, \qquad \psi_b \text{ a local minimum of } V_{\rm eff}(\psi),\ b \in \mathcal{B}. \tag{8}$$

The values $\{\psi_b\}$ are the *coherence vacuum branches*; symbols such as ψ_i or ψ_j in other documents are to be understood as elements of the same set $\mathcal{B}$ unless stated otherwise.

The physical coherence density obeys

$$C_{\rm phys}(x) = C_\star\, \psi(x), \tag{9}$$

and, in coarse–grained descriptions, may be decomposed into branch–resolved pieces

$$C_{\rm phys}(x) = \sum_{b\in\mathcal{B}} C^{(b)}_{\rm phys}(x), \qquad C^{(b)}_{\rm phys}(x) \geq 0, \tag{10}$$

which are required to satisfy this sum rule. The coherence contribution to the energy density is written analogously as

$$\rho_{\rm coh}(x) = \sum_{b\in\mathcal{B}} \rho_{{\rm coh},b}(x), \tag{11}$$

with $\rho_{{\rm coh},b}(x)$ denoting the effective energy density carried by branch b (for example, the corresponding part of $T^{(\rm coh)}_{00}$ in gravitational applications).

In practice, many branches may exist in principle, but on any given physical scale only a small subset is dynamically relevant. We denote this *effective branch set* by

$$\mathcal{B}_{\rm eff} \subseteq \mathcal{B}, \tag{12}$$

defined as the collection of branches whose contribution to the relevant observable (such as $C_{\rm phys}$ or $\rho_{\rm coh}$) exceeds a prescribed small fraction of the total. Branches outside $\mathcal{B}_{\rm eff}$ are treated as integrated out, with their effect absorbed into small renormalisations of the parameters associated with the active branches.

For completeness we also allow a branch–resolved decomposition of the coherence flux and source terms that appear in the gravity and jet documents. We write

$$J^i_C(x) = \sum_{b\in\mathcal{B}} J^i_{C,b}(x), \qquad S_C(x) = \sum_{b\in\mathcal{B}} S_{C,b}(x), \tag{13}$$

where $J^i_{C,b}$ and $S_{C,b}$ denote, respectively, the flux and source associated with branch b in a given effective description. Unless otherwise stated, symbols without a branch index refer to the total (branch–summed) quantities.

4 Taiji Spinor Parametrisation

4.1 Spinor representation of the SU(2) field

The $SU(2)$ field $U(x)$ may be parametrised in terms of a Taiji spinor $Z(x) \in \mathbb{C}^2$ subject to an appropriate normalisation condition. A standard construction associates to each normalised spinor Z an $SU(2)$ matrix $U(Z)$, for example via

$$U(Z) = \begin{pmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{pmatrix}, \qquad Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \qquad Z^\dagger Z = 1. \tag{14}$$

In the Taiji interpretation, $Z(x)$ encodes Excitation–Potential Field structure in a compact form, with $U(x)$ and the currents $L_\mu(x)$ derived from $Z(x)$.

4.2 Spinor norm as a parametrisation of ψ

The spinor representation introduces a natural positive scalar, the spinor norm $|Z(x)|^2 = Z^\dagger(x)Z(x)$. To connect this with the coherence order parameter $\psi(x)$, one introduces a reference spinor amplitude $Z_\star$ and writes

$$\psi(x) = \frac{C_{\mathrm{cl}}(x)}{C_\star} = \frac{|Z(x)|^2}{Z_\star^2}, \tag{15}$$

in regimes where $|Z|^2$ provides a faithful parametrisation of the expectation value $C_{\mathrm{cl}}(x)$. This relation defines how the spinor norm is to be interpreted: it is a *representation* of the classical coherence field $\psi(x)$, not a separate definition of coherence.

In particular:

- the primary coherence quantity is still $\hat{C}_R(x)$ and its expectation value $C_{\mathrm{cl}}(x)$,
- the spinor norm $|Z|^2$ is used to implement $\psi(x)$ in a convenient coordinate system,
- different choices of $Z_\star$ correspond to different normalisation conventions but do not alter the underlying coherence physics.

5 Energy Density, Coherence Density, and Normalisation

5.1 Coherence density and units

The dimensionful coherence density associated with $\psi(x)$ is defined as

$$C_{\mathrm{phys}}(x) = C_\star\, \psi(x) = C_{\mathrm{cl}}(x). \tag{16}$$

By construction, this quantity has the same dimension as the operator $\hat{C}_R(x)$ and therefore the same engineering dimension as an energy density. In the effective classical theory, $C_{\mathrm{phys}}(x)$ can be regarded as a measure of how strongly Excitation-induced order is realised in the local configuration.

5.2 Relation to energy density in calibrated regimes

In certain static, coarse-grained regimes, it is convenient to impose a calibration condition that relates the coherence density $C_{\mathrm{phys}}(x)$ to the local energy density $\epsilon(x) = T_{00}(x)$:

$$C_{\mathrm{phys}}(x) \simeq \alpha_C \, \epsilon(x), \tag{17}$$

where α_C is a dimensionless calibration constant. This relation is *not* taken as a fundamental identity; rather, it is used to fix the numerical values of $C_\star$ and α_C in environments where the relation is expected to hold approximately (for example, in a reference vacuum branch or in homogeneous matter distributions).

The calibration (17) has two roles:

- It ties the coherence scale $C_\star$ to physically measurable quantities, ensuring that $\psi(x)$ assumes values of order unity in relevant regimes.
- It provides a bridge between coherence-based descriptions and more conventional energy-density-based treatments, without collapsing coherence into "just energy density".

5.3 Coherence contribution to gravity

The gravitational sector is formulated in terms of the stress–energy tensor $T_{\mu\nu}$ entering the Einstein equations

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{18}$$

The coherence sector contributes to $T_{\mu\nu}$ via the energy density and pressure associated with $\psi(x)$ and its potential $V_{\mathrm{eff}}(\psi)$. There is no separate "coherence force" beyond this: gravity responds only to the total stress–energy.

In particular, the "total stress–energy" here is understood to include all sectors of the Taiji–Skyrme framework that carry energy and momentum. Schematically one may write

$$T^{\mathrm{tot}}_{\mu\nu} = T^{(\mathrm{Sk})}_{\mu\nu} + T^{(\mathrm{coh})}_{\mu\nu} + T^{(\mathrm{EM})}_{\mu\nu} + T^{(\mathrm{other})}_{\mu\nu},$$

where $T^{(\mathrm{Sk})}_{\mu\nu}$ denotes the SU(2) Skyrme contribution, $T^{(\mathrm{coh})}_{\mu\nu}$ collects the effective coherence terms (including spinor and covariant-derivative pieces), $T^{(\mathrm{EM})}_{\mu\nu}$ comes from the Maxwell sector associated with the internal $U(1)$ connection, and $T^{(\mathrm{other})}_{\mu\nu}$ covers any additional matter or dark components used in specific applications. The electromagnetic contribution has the usual Maxwell form

$$T^{(\mathrm{EM})}_{\mu\nu} = \frac{1}{g^2_{\mathrm{EM}}} \left(F_{\mu\alpha} F_{\nu}{}^{\alpha} - \frac{1}{4} \, g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right),$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and electromagnetic energy density $\rho_{\mathrm{EM}} \equiv T^{(\mathrm{EM})}_{00}$. For later use it is convenient to denote the total energy density by $\rho_{\mathrm{tot}}(x) \equiv T^{\mathrm{tot}}_{00}(x)$; when this quantity is employed as a gravitational source in phenomenological documents, it is understood to include ρ_{EM} alongside baryonic and coherence contributions unless an explicit approximation $\rho_{\mathrm{EM}} \simeq 0$ is stated.

Concretely, in the classical limit, the coherence field obeys an equation of motion derived from an effective Lagrangian of the form

$$\mathcal{L}_{\text{coh}} = \frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - V_{\text{eff}}(\psi) - \alpha\,\psi\,K(U), \tag{19}$$

and its contribution to $T_{\mu\nu}$ is obtained by functional variation with respect to the metric. The resulting energy density and pressure are then interpreted in terms of C_{phys} and ψ using the definitions above.

6 Coherence Usage Across the Taiji–Skyrme Documents

6.1 Symbol mapping

To prevent ambiguity, this subsection lists how the main coherence-related symbols used in the document suite map onto the canonical chain (2). The mapping is summarised in Table 1.

Although Table 1 focuses on coherence-related symbols, the Taiji–Skyrme framework also contains a Maxwell sector tied to the internal $U(1)$ phase of the Taiji spinor. Its basic objects follow the conventions of the Lagrangian/Hamiltonian document and are summarised here for reference:

- $A_\mu(x)$ is the $U(1)$ gauge potential ("coherence connection") associated with local phase rotations of the normalised spinor $\hat{Z}(x)$. Under $\hat{Z}(x) \to e^{i\lambda(x)}\hat{Z}(x)$ one has
$$A_\mu(x) \to A_\mu(x) + \partial_\mu\lambda(x).$$
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength. In any chosen frame, the electric and magnetic fields are encoded as $E_i = F_{0i}$ and $B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$.
- The covariant derivative
$$D_\mu\hat{Z} = (\partial_\mu - iqA_\mu)\,\hat{Z}$$
implements minimal coupling between the coherence spinor and the $U(1)$ connection, with q an effective coherence charge.
- The Maxwell Lagrangian
$$L_{\text{EM}} = -\frac{1}{4g_{\text{EM}}^2}F_{\mu\nu}F^{\mu\nu},$$
where g_{EM} is the electromagnetic coupling constant used throughout the Taiji–Skyrme framework and the coherence–connection term
$$L_{\hat{Z},A} = \kappa\,(D_\mu\hat{Z})^\dagger D^\mu\hat{Z} - V_{\hat{Z}}(\hat{Z})$$
together generate the sectoral stress–energy tensors $T_{\mu\nu}^{(\text{EM})}$ and $T_{\mu\nu}^{(\hat{Z},A)}$ by functional variation with respect to the metric.

These symbols are not independent definitions of coherence; they are the electromagnetic and covariant structures through which the coherence hierarchy couples to the Maxwell sector and, via $T_{\mu\nu}^{\text{tot}}$, to gravity.

Document	Symbols used	Interpretation in terms of the coherence chain
Quantum coherence	$\hat{C}_R(x)$	Primary coherence operator, defined as a renormalised composite of currents.
	$C_{\rm cl}(x)$, $\langle \hat{C}_R \rangle$	Classical coherence field $C_{\rm cl}(x) = \langle \hat{C}_R(x) \rangle$.
Taiji Skyrme Mathematical framework	$Z(x)$, $\lvert Z(x)\rvert^2$	Taiji spinor and its norm. In the coherence dictionary: $\psi(x) = \lvert Z(x)\rvert^2 / Z_\star^2$.
Normalisation / scaling	$\psi(x)$	Dimensionless coherence field defined by $\psi = C_{\rm cl}/C_\star$.
	$C_{\rm phys}(x)$	Coherence density $C_{\rm phys} = C_\star \psi$.
	α_C	Calibration constant in $C_{\rm phys} \simeq \alpha_C\, \epsilon$.
Unified particle / soliton structure	ψ_i, branch labels	Values of the coherence field ψ at vacuum branches.
	$\bar{\rho}_i$	Coarse-grained densities associated with each branch, constructed from $T_{00}[U, \psi]$.
Vacuum branches and domain walls	$\psi(x)$	Coherence field entering $V_{\rm eff}(\psi)$ and domain wall solutions, as defined above.
Galaxy phenomenology	$\rho_{\rm coh}(x)$	Effective matter density built from T_{00} of the coherence sector; parametrised ultimately by $\psi(x)$.
Cosmology	$\psi(t)$	Homogeneous coherence field in FRW; same ψ as in the dictionary.
	ρ_ψ, p_ψ	Energy density and pressure derived from ψ and $V_{\rm eff}(\psi)$.
Collapse and catastrophe	$\psi(t)$	Reduced dynamical variable for coherence, obeying overdamped or underdamped evolution equations derived from $V_{\rm eff}(\psi)$.

Table 1: Mapping of coherence-related symbols in the Taiji–Skyrme document suite to the canonical hierarchy (2).

6.2 Usage rules

The following rules govern the use of coherence in the Taiji–Skyrme framework:

1. The only fundamental coherence object at the quantum level is the operator $\hat{C}_R(x)$.
2. The classical coherence field is always $C_{\rm cl}(x) = \langle \hat{C}_R(x) \rangle$.
3. The dimensionless coherence field $\psi(x)$ is always defined by $\psi = C_{\rm cl}/C_\star$.
4. The spinor norm $|Z(x)|^2$ may be used to represent $\psi(x)$ via $\psi = |Z|^2/Z_\star^2$, but this is a parametrisation, not a separate definition.
5. The coherence density $C_{\rm phys}(x)$ is identified with $C_{\rm cl}(x)$ and has the same dimension as an energy density.
6. Any relation of the form $C_{\rm phys} \simeq \alpha_C\, T_{00}$ is a calibration condition in a specified regime, not a universal identity.
7. Gravity responds only to the total stress–energy tensor; there is no independent coherence-based force law.
8. When the term "coherence field" is used without qualification, it always refers to the classical order parameter $\psi(x)$ (or equivalently $C_{\rm cl}(x)$) and its effective dynamics. Electromagnetic quantities are written explicitly in terms of A_μ, $F_{\mu\nu}$, or $\rho_{\rm EM}$. In particular, references to "fields" in this dictionary do not introduce any separate, non-GR "coherence field" acting as a gravitational force beyond its contribution to $T^{\rm tot}_{\mu\nu}$.

7 Summary and Outlook

This document establishes a unified coherence framework for the Taiji–Skyrme theory. The key points are:

- Coherence is fundamentally a quantum operator $\hat{C}_R(x)$ built from Skyrme currents.
- The classical coherence field $C_{\rm cl}(x)$ is defined as the expectation value $\langle \hat{C}_R(x) \rangle$ in Excitation-selected states.
- The dimensionless coherence order parameter $\psi(x)$ is obtained by normalising $C_{\rm cl}(x)$ with a fixed reference scale $C_\star$.
- The Taiji spinor $Z(x)$ and its norm $|Z(x)|^2$ provide a convenient parametrisation of $\psi(x)$ but do not redefine coherence.
- The coherence density $C_{\rm phys}(x)$ is identified with $C_{\rm cl}(x)$ and can, in calibrated regimes, be related to the local energy density $T_{00}(x)$.
- Coherence influences gravity only through its contribution to the stress–energy tensor; no additional force law is introduced.

All existing and future Taiji–Skyrme documents are to interpret coherence-related symbols in terms of the chain

$$U, L_\mu \longrightarrow \hat{C}_R(x) \longrightarrow C_{\mathrm{cl}}(x) \longrightarrow \psi(x) \longrightarrow C_{\mathrm{phys}}(x), \rho_{\mathrm{coh}}(x). \tag{20}$$

This removes the ambiguity associated with multiple historical coherence definitions and provides a clear interface between the quantum, classical, and phenomenological levels of the theory.