Skyrme–Taiji Mathematical Framework

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Abstract

This document provides the classical mathematical backbone of the Skyrme–Taiji sector within the Taiji–Skyrme framework. The dynamical variable is an $SU(2)$ field $U(x)$ defined on a vacuum manifold topologically equivalent to S^3 and organised into topological sectors labelled by an integer winding number Q. The Taiji spinor $Z(x)$ and its norm $C(x) = |Z(x)|^2$ appear only at a structural level: they justify the use of $SU(2)$ as the configuration space and supply a classical coherence amplitude that can be linked, in other documents, to the full coherence dictionary.

We collect in a compact form the field definitions, configuration space, Skyrme–type Lagrangian, static energy functional, topological charge, hedgehog ansatz, and basic mass–radius scaling relations $M \sim f/e$, $R \sim 1/(ef)$. Quantum coherence operators, multi-vacuum structure, domain walls, galaxy phenomenology, cosmology, and catastrophe dynamics are not rederived here; they are treated in the specialised coherence, vacuum, and phenomenology texts that build on the classical structures summarised in this framework.

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1 Introduction

The purpose of this document is to provide a clean and compact mathematical backbone for the classical Skyrme–Taiji model. It focuses on:

- the field variables and configuration space,
- the SU(2) Skyrme–type Lagrangian and its symmetries,
- the associated topological charge,
- the reduction to spherically symmetric (hedgehog) configurations,
- the resulting radial energy functional and basic scaling laws,
- the internal $U(1)$ geometry of the Taiji spinor and its coherence connection A_μ.

No new physical hypotheses are introduced here. Quantum aspects of coherence, canonical quantisation, and Finkelstein–Rubinstein (FR) sectors are developed separately in the quantum coherence document, and the effective coherence and phenomenological constructions are treated in the dedicated coherence and parameter-organisation texts.

Here we restrict attention to the classical field theory that underlies all of these structures, and we keep coherence information at the level of:

1. the unified Taiji spinor $Z(x)$ and its norm,
2. the SU(2) field $U(x)$ built from Z,
3. the classical energy density T_{00} and its dependence on the Skyrme parameters.

In the full Taiji–Skyrme framework, this internal $U(1)$ structure is promoted to a Maxwell sector: the coherence connection A_μ is treated as an electromagnetic gauge potential with its own kinetic term and stress–energy tensor, as formulated in the Lagrangian/Hamiltonian document. The classical field content of the complete model then consists of the Skyrme sector $U(x)$, the coherence sector $(Z(x), \hat{Z}(x))$, and the Maxwell sector $A_\mu(x)$, whose combined stress–energy tensor $T^{\text{tot}}_{\mu\nu}$ will act as the source of gravity in the coherence–gravity texts. The present document isolates the purely classical Skyrme backbone together with the internal $U(1)$ geometry required for that coupling.

2 Field Variables and Configuration Space

2.1 The Taiji Spinor and SU(2) Field

The Taiji framework organises the Excitation–like and Potential Field–like components into a single complex spinor field

$$Z(x) = \begin{pmatrix} \Psi(x) \\ \Phi(x) \end{pmatrix}, \qquad \Psi(x), \Phi(x) \in \mathbb{C}, \tag{1}$$

defined at each spacetime point $x^\mu = (t, \boldsymbol{x})$. The spinor norm

$$C(x) \equiv |Z(x)|^2 = Z^\dagger(x) Z(x) = |\Psi(x)|^2 + |\Phi(x)|^2 \geq 0 \tag{2}$$

plays the role of a classical *coherence amplitude*: it measures how strongly the two components are present and aligned at x. In regions where $C(x)$ is close to its vacuum value, the internal order is high; deviations encode excitations and defects.

For the purpose of constructing the Skyrme–type dynamics, it is convenient to work with a unit-norm spinor

$$\hat{Z}(x) = \frac{Z(x)}{\sqrt{C(x)}}, \qquad \hat{Z}^\dagger(x)\hat{Z}(x) = 1, \tag{3}$$

whenever $C(x) > 0$. From $\hat{Z}(x)$, one can build an SU(2) group element $U(x)$ that parametrises the vacuum manifold:

$$U(x) \in \mathrm{SU}(2), \qquad U(x)U^\dagger(x) = \mathbf{1}, \quad \det U(x) = 1. \tag{4}$$

The explicit map $\hat{Z} \mapsto U$ is not needed here; it is enough that:

- $\hat{Z}$ lives on a three-sphere S^3,
- SU(2) is topologically S^3,
- there exists a smooth, bijective identification between these manifolds.

We will therefore treat $U(x)$ as the *primary* field variable in the classical Skyrme–Taiji Lagrangian, with the spinor $Z(x)$ and the amplitude $C(x)$ providing the coherence interpretation.

2.2 Coherence Internal Space and Local Phase Redundancy

In the previous subsection we introduced the Taiji spinor

$$Z(x) = \begin{pmatrix} \Psi(x) \\ \Phi(x) \end{pmatrix}, \qquad x^\mu = (t, \mathbf{x}),$$

with classical coherence amplitude

$$C(x) \equiv |Z(x)|^2 = Z^\dagger(x)Z(x) = |\Psi(x)|^2 + |\Phi(x)|^2 \geq 0.$$

Whenever $C(x) > 0$ we defined the unit–norm spinor

$$\hat{Z}(x) \equiv \frac{Z(x)}{\sqrt{C(x)}}, \qquad \hat{Z}^\dagger(x)\,\hat{Z}(x) = 1,$$

and used $\hat{Z}$ to construct the $SU(2)$ field $U(x)$ that furnishes the classical Skyrme dynamics.

At this level, $\hat{Z}$ is not unique. Multiplying $\hat{Z}$ at each spacetime point by a phase factor leaves both its norm and the associated $SU(2)$ element unchanged. Explicitly, for any real function $\lambda(x)$ we have the local phase transformation

$$\hat{Z}(x) \;\longrightarrow\; \hat{Z}'(x) = e^{i\lambda(x)}\,\hat{Z}(x), \tag{5}$$

under which

$$\hat{Z}'^\dagger\hat{Z}' = \hat{Z}^\dagger\hat{Z} = 1.$$

The map from $\hat{Z}$ to U is insensitive to this phase: different representatives $\hat{Z}$ and $e^{i\lambda(x)}\hat{Z}$ correspond to the same $SU(2)$ field $U(x)$. Thus the physically relevant internal coherence state is not the spinor $\hat{Z}$ itself, but its equivalence class under local phase rotations.

This identifies the internal coherence space with the complex projective line

$$\mathrm{CP}^1 \simeq S^2,$$

where each point corresponds to a ray $[\hat{Z}]$ rather than to a specific spinor representative. In particular, the classical alignment variables can be expressed as

$$n^a(x) \equiv \hat{Z}^\dagger(x)\,\sigma^a\,\hat{Z}(x), \qquad a = 1, 2, 3,$$

with

$$n^a n^a = 1,$$

so that $n^a(x)$ parametrises a point on the internal two–sphere S^2 at each spacetime point.

The transformation (5) is a *redundancy* in the description of the same physical coherence configuration. Two spinors related by a local phase represent the same internal direction $n^a(x)$ and the same $SU(2)$ field $U(x)$. We will therefore treat (5) as a local $U(1)$ gauge freedom in the coherence sector.

Coherence Connection as an Internal Gauge Field

Once the local phase redundancy has been identified, it is natural to introduce a connection that tracks how the coherence state twists in internal space as one moves through spacetime. A convenient choice is the one–form

$$A_\mu(x) \equiv -\,i\,\hat{Z}^\dagger(x)\,\partial_\mu \hat{Z}(x), \qquad \mu = 0,1,2,3, \tag{6}$$

which is real by construction. Under a local phase transformation (5),

$$\hat{Z}(x) \to \hat{Z}'(x) = e^{i\lambda(x)}\,\hat{Z}(x), \tag{7}$$

$$\partial_\mu \hat{Z}(x) \to \partial_\mu \hat{Z}'(x) = e^{i\lambda(x)}\big(\partial_\mu \hat{Z}(x) + i(\partial_\mu \lambda(x))\hat{Z}(x)\big), \tag{8}$$

so that

$$\begin{aligned} A'_\mu(x) &= -\,i\,\hat{Z}'^\dagger(x)\,\partial_\mu \hat{Z}'(x) \\ &= -\,i\,\hat{Z}^\dagger(x)\,\big(\partial_\mu \hat{Z}(x) + i(\partial_\mu\lambda(x))\hat{Z}(x)\big) \\ &= -\,i\,\hat{Z}^\dagger(x)\,\partial_\mu \hat{Z}(x) + \partial_\mu \lambda(x) \\ &= A_\mu(x) + \partial_\mu \lambda(x). \end{aligned} \tag{9}$$

Thus the coherence connection A_μ transforms as

$$A_\mu \to A_\mu + \partial_\mu \lambda,$$

which is precisely the $U(1)$ gauge transformation law satisfied by an electromagnetic vector potential.

We emphasise that at this stage A_μ is defined as a *composite* object built from $\hat{Z}$; no independent dynamics has yet been assigned to it. The identification (6)–(9) shows that the internal phase redundancy of the Taiji spinor naturally supports a $U(1)$ connection. In later sections this observation will be used to define a Maxwell-type dynamical sector in which A_μ becomes the electromagnetic field associated with coherence, and its curvature

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

will play the role of the classical electromagnetic field strength.

2.3 Maxwell and the Coherence Connection

Equations (6)–(9) show that the coherence connection $A_\mu(x)$ built from the normalized Taiji spinor $\hat{Z}(x)$ transforms under local phase rotations as

$$A_\mu \;\to\; A_\mu + \partial_\mu \lambda(x)\,,$$

which is precisely the $U(1)$ gauge transformation law of an electromagnetic vector potential. In this sense, the internal phase redundancy of the coherence spinor provides a natural geometric origin for a $U(1)$ gauge field.

In the present framework, the identification

$$A_\mu \sim -i\hat{Z}^\dagger \partial_\mu \hat{Z}$$

should be viewed as a *geometric realisation* of the coherence connection: for any given spinor configuration $\hat{Z}(x)$ one can construct a Berry-type connection with the correct $U(1)$ transformation law. When a dynamical electromagnetic sector is introduced, it is convenient to treat A_μ as an independent field carrying the same gauge transformation law, with its dynamics and stress–energy specified elsewhere.

The full Maxwell sector and its coupling to coherence are defined in the Lagrangian/Hamiltonian framework document. There:

- the same A_μ is endowed with a standard Maxwell kinetic term and contributes a stress–energy tensor $T^{(\mathrm{EM})}_{\mu\nu}$,
- the covariant derivative $D_\mu \hat{Z} = (\partial_\mu - iqA_\mu)\hat{Z}$ implements minimal coupling of the coherence spinor to the connection,
- the electromagnetic and coherence contributions are combined with the Skyrme sector into a single total stress–energy tensor $T^{\mathrm{tot}}_{\mu\nu}$ which will source gravity in the global Taiji–Skyrme framework.

The role of the present document is therefore limited to establishing the internal $U(1)$ geometry and the existence of a natural coherence connection A_μ. All dynamical details of the Maxwell sector—including the explicit form of the Maxwell Lagrangian, canonical structure, and $T^{(\mathrm{EM})}_{\mu\nu}$— are collected in the Lagrangian/Hamiltonian companion text.

2.4 Finite-Energy Configurations and Topology

We restrict attention to static configurations $U(\boldsymbol{x})$ in three-dimensional space, with the energy functional

$$E[U] = \int_{\mathbb{R}^3} \mathrm{d}^3x \; \mathcal{E}[U(\boldsymbol{x})], \tag{10}$$

where $\mathcal{E}$ is the static energy density derived from the Lagrangian.

Finite energy requires that $U(\boldsymbol{x}) \to U_\infty$ as $|\boldsymbol{x}| \to \infty$, where U_∞ is a constant vacuum value. By a global SU(2) rotation we can take $U_\infty = \mathbf{1}$, so finite-energy configurations define maps

$$U : S^3 \to S^3, \tag{11}$$

after one-point compactification of $\mathbb{R}^3$ to S^3. These maps are classified by the homotopy group

$$\pi_3(S^3) \cong \mathbb{Z}, \tag{12}$$

so that each finite-energy configuration belongs to a topological sector labelled by an integer $Q \in \mathbb{Z}$. This integer is the *topological charge* or *winding number*, interpreted as a Skyrmion number.

A standard expression for Q in terms of $U(\boldsymbol{x})$ is

$$Q[U] = -\frac{1}{24\pi^2} \int_{\mathbb{R}^3} \epsilon_{ijk} \, \mathrm{Tr}(L_i L_j L_k) \; \mathrm{d}^3x, \tag{13}$$

where the left-invariant currents are defined below.

3 Skyrme–Taiji Lagrangian and Symmetries

3.1 Left-Invariant Currents

The basic building blocks of the Skyrme–type Lagrangian are the left-invariant currents

$$L_\mu = U^\dagger \partial_\mu U, \qquad \mu = 0, 1, 2, 3. \tag{14}$$

These take values in the Lie algebra $\mathfrak{su}(2)$ and can be expanded as

$$L_\mu = i L_\mu^a \sigma_a, \qquad a = 1, 2, 3, \tag{15}$$

with σ_a the Pauli matrices and L_μ^a real components.

The currents transform covariantly under global SU(2) rotations:

$$U(x) \mapsto G U(x), \qquad G \in \mathrm{SU}(2) \text{ constant } \Rightarrow \; L_\mu \mapsto L_\mu, \tag{16}$$

so any Lagrangian constructed from traces of products of L_μ is automatically invariant under this internal symmetry.

3.2 Skyrme–Taiji Lagrangian

The classical Skyrme–Taiji Lagrangian density for the field $U(x)$ is taken to be

$$\mathcal{L} = \frac{f^2}{4} \operatorname{Tr}\Big(\partial_\mu U^\dagger \partial^\mu U\Big) + \frac{1}{32e^2} \operatorname{Tr}\Big([U^\dagger \partial_\mu U,\, U^\dagger \partial_\nu U]^2\Big) - V(U). \tag{17}$$

Here:

- f has dimensions of energy,
- e is dimensionless,
- $V(U)$ is a potential term whose minima define the vacuum manifold.

The first term is the nonlinear sigma-model kinetic term, measuring how rapidly U varies in spacetime. The second is the Skyrme term, which stabilises finite-energy solitons by penalising large gradients and preventing collapse. Together they support topologically nontrivial, stable configurations with finite energy.

In terms of the left-invariant currents (14), the Lagrangian can be written as

$$\mathcal{L} = -\frac{f^2}{4} \operatorname{Tr}(L_\mu L^\mu) + \frac{1}{32e^2} \operatorname{Tr}\big([L_\mu, L_\nu]^2\big) - V(U), \tag{18}$$

using the unitarity of U.

3.3 Symmetries

The Lagrangian (17) is invariant under:

- Global SU(2) rotations $U \mapsto GU$, $G \in \mathrm{SU}(2)$,
- Poincaré transformations in spacetime,
- Parity $P : \boldsymbol{x} \mapsto -\boldsymbol{x}$ with an appropriate action on U,
- Time reversal T with complex conjugation as needed.

The global SU(2) symmetry leads to a conserved Noether current, and Poincaré invariance leads to a conserved, symmetric stress–energy tensor.

The detailed canonical Hamiltonian and Poisson structure associated with (17) are treated in the separate Hamiltonian document; here we focus on the equations of motion and the static energy functional.

4 Equations of Motion and Energy Functional

4.1 Euler–Lagrange Equations

Varying the action

$$S[U] = \int \mathrm{d}^4x\, \mathcal{L}[U] \tag{19}$$

with respect to U under the constraint $U \in \mathrm{SU}(2)$ yields the equations of motion. A standard way to express the result is

$$\partial_\mu \left(f^2 L^\mu - \frac{1}{4e^2}[L_\nu, [L^\mu, L^\nu]] \right) + \frac{\partial V}{\partial U} U^\dagger - U \left(\frac{\partial V}{\partial U} \right)^\dagger = 0, \tag{20}$$

which is an $\mathfrak{su}(2)$-valued equation. The detailed form of the potential contribution depends on the specific choice of $V(U)$, but the structure of the kinetic and Skyrme terms is fixed.

4.2 Static Energy Density

For static fields $U(\boldsymbol{x})$, the time derivatives vanish and the energy density $\mathcal{E}$ is obtained from $-\mathcal{L}$ (with appropriate sign choice) as

$$\mathcal{E}[U] = -\mathcal{L}\big|_{\partial_0 U = 0} = \frac{f^2}{4} \operatorname{Tr}(L_i L_i) - \frac{1}{32e^2} \operatorname{Tr}\big([L_i, L_j]^2\big) + V(U), \tag{21}$$

where $i, j = 1, 2, 3$ label spatial components and repeated indices are summed.

The total energy is

$$E[U] = \int_{\mathbb{R}^3} \mathrm{d}^3x\, \mathcal{E}[U(\boldsymbol{x})]. \tag{22}$$

Solitonic solutions correspond to finite-energy stationary points of this functional within a fixed topological sector Q.

5 Hedgehog Ansatz and Radial Reduction

5.1 Hedgehog Form

To make the structure of solitons explicit and to obtain tractable equations, we consider spherically symmetric (hedgehog) configurations of the form

$$U(\boldsymbol{x}) = \exp(iF(r)\, \hat{\boldsymbol{x}} \cdot \boldsymbol{\sigma}) = \cos F(r)\, \mathbf{1} + i \sin F(r)\, \hat{\boldsymbol{x}} \cdot \boldsymbol{\sigma}, \tag{23}$$

where

$$r = |\boldsymbol{x}|, \qquad \hat{\boldsymbol{x}} = \frac{\boldsymbol{x}}{r}, \qquad F(r) \in \mathbb{R} \tag{24}$$

is the radial profile function and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$.

Finite energy and unit topological charge $Q = 1$ are obtained by the boundary conditions

$$F(0) = \pi, \qquad F(r) \to 0 \quad \text{as} \quad r \to \infty. \tag{25}$$

These conditions ensure that $U(\boldsymbol{x})$ wraps the vacuum manifold once as r runs from 0 to ∞, yielding $Q = 1$.

5.2 Radial Energy Functional

For the hedgehog (23), the static energy (21) reduces to a one-dimensional functional of $F(r)$. After standard manipulations, one finds

$$E[F] = 4\pi \int_0^\infty \mathrm{d}r\, r^2\, \mathcal{E}_{\mathrm{rad}}(r), \tag{26}$$

where the radial energy density is

$$\mathcal{E}_{\mathrm{rad}}(r) = \frac{f^2}{2}\left(F'^2 + \frac{2\sin^2 F}{r^2}\right) + \frac{1}{2e^2}\left(\frac{\sin^2 F}{r^2}F'^2 + \frac{\sin^4 F}{2r^4}\right) + V(F). \tag{27}$$

Here $F' = \mathrm{d}F/\mathrm{d}r$, and $V(F)$ denotes the potential expressed in terms of the hedgehog profile. The exact form of $V(F)$ depends on the chosen symmetry-breaking pattern in the vacuum sector.

The Euler–Lagrange equation for $F(r)$ is obtained by varying (27) with respect to F:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2\left(f^2F' + \frac{2}{e^2}\frac{\sin^2 F}{r^2}F'\right)\right] - 2f^2\sin F\cos F - \frac{2}{e^2}\frac{\sin^3 F\cos F}{r^2} + r^2\frac{\mathrm{d}V}{\mathrm{d}F} = 0. \tag{28}$$

Solitonic profiles $F(r)$ are solutions of (28) satisfying the boundary conditions (25).

6 Dimensionless Variables and Scaling

6.1 Radial Rescaling

It is convenient to absorb the parameters f and e into a dimensionless radial coordinate. Introduce

$$\rho = ef\, r, \tag{29}$$

so that

$$r = \frac{\rho}{ef}, \qquad \frac{\mathrm{d}}{\mathrm{d}r} = ef\frac{\mathrm{d}}{\mathrm{d}\rho}, \qquad F'(r) = ef\, F_\rho(\rho), \tag{30}$$

where $F_\rho \equiv \mathrm{d}F/\mathrm{d}\rho$. Substituting into (27) yields

$$\begin{aligned} E[F] &= 4\pi\int_0^\infty \mathrm{d}r\, r^2\, \mathcal{E}_{\mathrm{rad}}(r) \\ &= 4\pi\int_0^\infty \frac{\mathrm{d}\rho}{ef}\left(\frac{\rho}{ef}\right)^2\left\{\frac{f^2}{2}\left[(ef)^2F_\rho^2 + \frac{2\sin^2 F}{r^2}\right] + \frac{1}{2e^2}\left[\frac{\sin^2 F}{r^2}(ef)^2F_\rho^2 + \frac{\sin^4 F}{2r^4}\right] + V(F)\right\}. \end{aligned} \tag{31}$$

Using $r = \rho/(ef)$ and simplifying, one obtains the standard scaling form

$$E[F] = \frac{f}{e}\,\mathcal{I}[F;\tilde{V}], \tag{32}$$

where $\mathcal{I}$ is a dimensionless functional of $F(\rho)$ and of the dimensionless potential $\tilde{V}$ obtained from V by factoring out f^4 (or another suitable scale).

The precise expression for $\mathcal{I}$ depends on the chosen potential, but the key point is that:

- all dependence on the absolute scales f and e factorises as f/e;
- the shape of the soliton is determined by the dimensionless equation of motion in ρ, which depends only on $\tilde{V}$.

6.2 Mass and Radius Scaling

If we define a characteristic dimensionless size ρ_0 of the soliton (for instance where $F(\rho_0) = \pi/2$ or via a radius containing a fixed fraction of the energy), then the physical radius scales as

$$R \sim \frac{\rho_0}{ef}, \tag{33}$$

while the energy scales as (32), $E \sim f/e$.

Thus, at the classical level:

- increasing f at fixed e increases both the energy and decreases the size,
- increasing e at fixed f decreases both the energy and the size.

More refined mass–radius relations, including coherence-dependent effective densities and gravitational coupling, are developed in the structural parameter and phenomenology documents.

7 Classical Coherence Measures

7.1 Spinor Amplitude and Coherence Density

The classical coherence amplitude is given by the spinor norm (2), $C(x) = |Z(x)|^2$. To construct a dimensionless measure of coherence, one introduces a reference scale $v > 0$ and defines

$$\psi(x) = \frac{C(x)}{v^2}, \tag{34}$$

which provides a scale-free indicator of how strongly the internal Taiji spinor is aligned relative to the reference scale.

In effective descriptions, a physical coherence density C_{phys} with the dimensions of an energy density is obtained by introducing a fixed density scale $C_\star$ and writing

$$C_{\mathrm{phys}}(x) = C_\star\, \psi(x). \tag{35}$$

The relation between C_{phys} and the classical energy density T_{00} is treated phenomenologically in the coherence normalisation and vacuum-structure documents; here we simply note that $C(x)$ and $\psi(x)$ provide the bridge between the Taiji spinor $Z(x)$ and the classical Skyrme field $U(x)$.

7.2 Role Within the Classical Framework

Within the strictly classical Skyrme–Taiji model described here:

- $U(x)$ is the dynamical field entering the Lagrangian,
- $Z(x)$ and its norm $C(x)$ supply the coherence interpretation and the link to Excitation–Potential Field structure,
- the local energy density $T_{00}[U]$ and the soliton profiles $F(r)$ determine how coherence is distributed in space.

Quantum coherence operators, uncertainty relations, and entanglement measures are defined only at the quantum level and are not part of this purely classical framework.

7.3 Multi-Vacuum Structure and Coherence Branches

The dimensionless coherence field $\psi(x)$ introduced above can be treated, at a coarse-grained level, as an effective scalar degree of freedom with its own potential energy density $V_{\rm eff}(\psi)$. The detailed form of this potential and its parameter dependence are developed in the dedicated vacuum-structure document; here we record only the structural features that will be used throughout the Taiji–Skyrme framework.

We assume that $V_{\rm eff}(\psi)$ admits a finite set of local minima separated by barriers,

$$\left.\frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}\psi}\right|_{\psi=\psi_b} = 0, \qquad \left.\frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}\psi^2}\right|_{\psi=\psi_b} > 0, \qquad b = 1, \ldots, N, \tag{36}$$

where the real numbers $\{\psi_b\}_{b=1}^{N}$ label the positions of the coherence vacuum branches. Each branch b corresponds to a quasi-vacuum sector in which $\psi(x)$ fluctuates around ψ_b. In the vicinity of such a minimum we write

$$\psi(x) = \psi_b + \delta\psi_b(x), \tag{37}$$

and the small fluctuations $\delta\psi_b$ are governed by an effective mass parameter

$$m_b^2 \;\equiv\; \left.\frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}\psi^2}\right|_{\psi=\psi_b}, \tag{38}$$

as it appears in the reduced scalar Lagrangian for ψ developed in the vacuum-structure text. The explicit kinetic and interaction terms for ψ are not needed in the present framework; only the existence of these minima and the associated mass scales will be used.

The integer N should be understood as a structural upper bound: in principle the effective coherence sector may admit many branches, but on any given physical scale only a small subset will be dynamically relevant. We will therefore distinguish between

- the full set of branches $\mathcal{B} = \{1, 2, \ldots, N\}$, and
- the effective branch set $\mathcal{B}_{\rm eff}$ appropriate to a given problem (for example, a handful of branches on galactic scales, or a different subset in early-universe cosmology).

Branches outside $\mathcal{B}_{\rm eff}$ are treated as either unpopulated or effectively integrated out, contributing only small renormalisations of the parameters associated with the active branches. This "many branches in principle, few effective branches per scale" principle will be used repeatedly in the vacuum-structure, collapse, gravity, galaxy-rotation, and cosmology documents, and its notation is fixed here at the level of the classical Skyrme–Taiji framework.

8 Summary

This document has collected the core classical ingredients of the Skyrme–Taiji model:

- the Taiji spinor $Z(x)$ and coherence amplitude $C(x) = |Z(x)|^2$,
- the SU(2) field $U(x)$ built from $\hat{Z}(x)$ and its left-invariant currents L_μ,
- the Skyrme–type Lagrangian (18) and its symmetries,
- the definition of topological charge Q via (13),
- the hedgehog ansatz and radial reduction to a single profile $F(r)$,
- the dimensionless scaling with parameters f and e, leading to $E \sim f/e$ and $R \sim 1/(ef)$.

These structures form the classical backbone on which the quantum coherence, vacuum structure, domain walls, cosmology, and phenomenological applications are built in the other Taiji–Skyrme documents.