Coherence Vacuum Structure and Domain Walls in the Taiji–Skyrme Framework

Effective Potential, Multi-Branch Coherence, and Interface Dynamics

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Abstract

The Taiji–Skyrme framework couples an $SU(2)$ field $U(x)$ to a coherence structure encoded in a scalar field derived from an underlying Taiji spinor. The normalisation of coherence and its relation to energy density have been formulated in terms of a dimensionless field $\psi(x)$ and a corresponding physical coherence density $C_{\text{phys}}(x) = C_\star \psi(x)$. In the present document we develop the vacuum structure and domain-wall dynamics of this coherence sector. We introduce an effective scalar Lagrangian for ψ, construct a multi-vacuum potential $V_{\text{eff}}(\psi)$, and analyse its minima, stability properties, and parameter dependence. The resulting vacuum branches are characterised by their coherence amplitudes and associated densities, while domain walls arise as static interfaces interpolating between neighbouring minima. We derive the field equation for ψ from an explicit effective Lagrangian, obtain the domain-wall profile in the standard first-integral form, and discuss how bifurcations of the vacuum structure emerge when control parameters are varied. Throughout, coherence is treated as a structural scalar degree of freedom that influences gravitational phenomena only through its contribution to the stress–energy tensor; no additional force mechanism beyond spacetime curvature is introduced.

1 Introduction

The Taiji–Skyrme theory is built upon an $SU(2)$ field $U(x)$ whose dynamics are governed by a Skyrme-type Lagrangian, together with a coherence structure derived from a Taiji spinor $Z(x) \in \mathbb{C}^2$. The spinor provides a scalar amplitude that measures the local degree of internal alignment, while the group-valued field U carries the topological and dynamical content. At large scales, this combined system exhibits multiple coherence branches, domain-like regions associated with different vacuum values, and interfaces where the coherence field transitions between branches.

A consistent description of this vacuum structure requires three elements:

1. a precise definition and normalisation of the coherence field,
2. an effective potential $V_{\text{eff}}(\psi)$ admitting multiple minima,
3. and a dynamical equation for ψ derived from an effective Lagrangian that couples it to the Skyrme sector.

These ingredients determine the possible vacuum branches, the energy cost of interpolating between them, and the stability of the resulting configurations under deformations of external or internal parameters.

The present document is devoted to this vacuum and interface problem. We treat the coherence field as a real scalar $\psi(x)$ with a canonically normalised kinetic term, subject to an effective potential $V_{\mathrm{eff}}(\psi)$ chosen to support multiple minima. The coupling to the Skyrme sector is modelled through an interaction between ψ and a scalar activity functional built from the left-invariant currents of U. From the resulting effective Lagrangian, we derive the field equation for ψ, determine the structure of the vacuum branches, and analyse static domain-wall solutions interpolating between neighbouring vacua.

The aim is to provide a field-theoretic description of coherence vacuum structure and domain walls that is:

- compatible with the established coherence normalisation and parameter mapping,
- explicit at the level of the effective Lagrangian and its Euler–Lagrange equations,
- and independent of any additional gravitational force law beyond general relativity.

The gravitational and phenomenological implications of the branch structure are addressed in separate analyses; here we focus on the internal structure of the coherence sector itself.

2 Coherence Variables and Normalisation

The Taiji spinor

$$Z(x) = \begin{pmatrix} \Psi(x) \\ \Phi(x) \end{pmatrix}, \qquad Z \in \mathbb{C}^2, \tag{1}$$

defines a scalar coherence amplitude through its norm

$$C(x) = |Z(x)|^2 = Z^\dagger(x) Z(x) = |\Psi(x)|^2 + |\Phi(x)|^2. \tag{2}$$

The quantity $C(x)$ is non-negative and invariant under global $SU(2)$ transformations acting on Z. It measures the local magnitude of the spinor and thus the degree of internal alignment of the underlying degrees of freedom.

To obtain a dimensionless coherence variable suitable for comparison across scales and regimes, we introduce a reference amplitude $v > 0$ and define

$$\psi(x) = \frac{C(x)}{v^2}. \tag{3}$$

The field $\psi(x)$ is dimensionless and normalised such that $\psi \sim \mathcal{O}(1)$ in typical configurations. It is this dimensionless coherence field that will appear in the effective potential $V_{\mathrm{eff}}(\psi)$ and in the classification of vacuum branches.

In order to connect coherence to energy density, we introduce a fixed reference scale $C_\star$ with dimension

$$[C_\star] = L^{-4},$$

and define the physical coherence density

$$C_{\rm phys}(x) = C_\star\,\psi(x). \tag{4}$$

The scale $C_\star$ is chosen such that $C_{\rm phys}$ can be compared directly with the energy density T_{00} stemming from the Skyrme and coherence sectors. In coarse-grained descriptions, it is often convenient to introduce an effective proportionality relation of the form

$$C_{\rm phys}(x) = \alpha_C\,T_{00}(x), \tag{5}$$

where α_C is a dimensionless or slowly varying parameter encoding how variations in the energy density are reflected in the coherence density. This relation is not a definition of either quantity; it is an effective correspondence valid in regimes where the medium can be treated as approximately homogeneous on the relevant scales.

The set of coherence parameters relevant for the present analysis is therefore

$$\{v,\ C_\star,\ \alpha_C\}, \tag{6}$$

together with the functional form of the effective potential $V_{\rm eff}(\psi)$ introduced below. These parameters determine the locations of vacuum branches, the associated coherence densities, and the energy cost of moving between branches.

3 Effective Coherence Lagrangian and Origin of $V_{\rm eff}(\psi)$

The microscopic dynamics of the Taiji–Skyrme field are governed by an $SU(2)$ Lagrangian constructed from the left-invariant currents

$$L_\mu = U^\dagger \partial_\mu U, \tag{7}$$

with a sigma-model term, a Skyrme term, and possible mass terms. To describe the vacuum structure and domain walls of the coherence sector, we supplement this with an effective scalar Lagrangian for the dimensionless field $\psi(x)$.

At the level of effective field theory, the simplest choice consistent with Lorentz invariance and a real scalar field is

$$\mathcal{L}_\psi = \frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - V_{\rm eff}(\psi), \tag{8}$$

where $V_{\rm eff}(\psi)$ is an effective potential admitting multiple minima. This potential encapsulates the self-interactions of the coherence field at the coarse-grained level and is assumed to be bounded from below to ensure stability.

To account for the influence of the Skyrme sector on coherence, and vice versa, we introduce an interaction term of the form

$$\mathcal{L}_{\rm int} = -\alpha\,\psi(x)\,K(U), \tag{9}$$

where α is a coupling constant and $K(U)$ is a scalar functional of the currents L_μ that measures the local "activity" of the Skyrme field. A natural choice is

$$K(U) = {\rm Tr}(L_\mu L^\mu), \tag{10}$$

which is Lorentz-invariant and dimensionally compatible with a scalar source. Other choices are possible, but the essential feature is that $K(U)$ increases where the gradients of U are large, signalling regions of high structural activity that can act as a source or sink for coherence.

The combined effective Lagrangian for the coherence and interaction sectors is then

$$\mathcal{L}_{\rm coh}[U,\psi] = \frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - V_{\rm eff}(\psi) - \alpha\,\psi\,K(U). \tag{11}$$

Variation with respect to ψ yields the Euler–Lagrange equation

$$\partial_\mu\partial^\mu\psi(x) + \frac{dV_{\rm eff}}{d\psi}(\psi(x)) - \alpha\,K(U(x)) = 0, \tag{12}$$

which governs the dynamics of the coherence field in the presence of the Skyrme background. This equation replaces any ad hoc postulated relations between coherence and Skyrme activity: it follows directly from the effective Lagrangian (11).

The form of $V_{\rm eff}(\psi)$ is constrained by several requirements:

- It must be bounded from below to ensure a stable ground state.
- It must admit multiple local minima to support distinct vacuum branches.
- Its structure should be compatible with the symmetries and scaling properties of the underlying theory.

A common choice is a polynomial potential whose coefficients are tuned to produce two or more minima separated by finite barriers. In subsequent sections we will specify a concrete class of potentials, analyse their minima and curvature properties, and study the resulting vacuum branches and domain-wall configurations.

The gravitational coupling of the coherence sector enters through the stress–energy tensor derived from (11), which contributes to the total $T_{\mu\nu}[U,\psi]$ sourcing the Einstein equations. No additional gravitational force term is introduced by the Lagrangian (11); coherence influences gravity exclusively through its effect on the energy density and pressures encoded in $T_{\mu\nu}$.

4 Multi-Vacuum Potential and Vacuum Branches

The effective potential $V_{\rm eff}(\psi)$ encodes the self-interactions of the coherence field at the coarse-grained level. To support multiple coherence branches, it must admit several local minima separated by finite barriers. Rather than postulating a single specific polynomial from the outset, we adopt a general class of potentials constructed around a chosen set of branch values.

Let $\{\psi_i\}_{i=1}^N$ be a set of real numbers labelling candidate coherence branch positions. A natural way to construct a potential with minima at (or near) these values is

$$V_{\rm eff}(\psi) = \lambda\prod_{i=1}^{N}\big(\psi-\psi_i\big)^2 + \Delta V(\psi), \tag{13}$$

where $\lambda > 0$ sets the overall stiffness of the potential, and $\Delta V(\psi)$ is a lower-degree polynomial used to lift degeneracies and adjust the relative depths of the minima. The product structure ensures that each ψ_i is a stationary point of the leading term, while ΔV allows controlled deformations without destroying boundedness from below.

The first derivative is

$$V'_{\text{eff}}(\psi) = 2\lambda \sum_{k=1}^{N} \left[(\psi - \psi_k) \prod_{\substack{i=1 \\ i \neq k}}^{N} (\psi - \psi_i)^2 \right] + \Delta V'(\psi), \tag{14}$$

and the second derivative is obtained by differentiating (14). At a putative vacuum position $\psi = \psi_j$ we find

$$V'_{\text{eff}}(\psi_j) = \Delta V'(\psi_j), \tag{15}$$

since all product terms vanish. Thus, in the simplest case where $\Delta V'(\psi_j) = 0$ for all j, the points ψ_j are exact stationary points of V_{eff}. The curvature at ψ_j is then

$$V''_{\text{eff}}(\psi_j) = 2\lambda \prod_{\substack{i=1 \\ i \neq j}}^{N} (\psi_j - \psi_i)^2 + \Delta V''(\psi_j). \tag{16}$$

Since $\lambda > 0$ and the squared factors are non-negative, a sufficient condition for ψ_j to be a local minimum is

$$V''_{\text{eff}}(\psi_j) > 0. \tag{17}$$

In practice, one often works with a small number of branches, such as $N = 2$ (double-well) or $N = 3$ (triple-well) potentials. The explicit form of $\Delta V(\psi)$ can then be chosen to tune the relative depths of the minima while maintaining $V_{\text{eff}}(\psi) \to +\infty$ as $|\psi| \to \infty$. For example, in a double-well case with target minima at ψ_1 and ψ_2, one may take

$$V_{\text{eff}}(\psi) = \lambda(\psi - \psi_1)^2(\psi - \psi_2)^2 + a_1\psi + a_0, \tag{18}$$

with coefficients (a_0, a_1) adjusted such that $V_{\text{eff}}(\psi_1)$ and $V_{\text{eff}}(\psi_2)$ have the desired depths. The generalisation to $N = 3$ is straightforward and follows the same logic.

Once the minima ψ_i have been specified, the associated physical coherence densities are

$$C^{(i)}_{\text{phys}} = C_\star \, \psi_i, \tag{19}$$

and, in regions where $\psi(x)$ is approximately constant and equal to ψ_i, the energy density approaches a branch-dependent value $\bar{\epsilon}_i$. The triplet

$$\left\{ \psi_i,\ C^{(i)}_{\text{phys}},\ \bar{\epsilon}_i \right\} \tag{20}$$

thus characterises each vacuum branch: ψ_i encodes the coherence amplitude, $C^{(i)}_{\text{phys}}$ its density representation, and $\bar{\epsilon}_i$ its energetic level. These quantities feed into the effective mass density and gravitational response via the total stress–energy tensor, but the detailed parameter mapping is handled elsewhere.

In what follows, we concentrate on the local structure near neighbouring minima and on the domain walls that interpolate between them. The global shape of V_{eff} can be more complicated, but the essential features of interfaces and bifurcations are determined by its behaviour in the vicinity of its extrema.

5 Domain Walls and Interface Profiles

Domain walls arise as static configurations of the coherence field that interpolate between distinct vacuum branches of the effective potential. In the Taiji–Skyrme framework, these interfaces separate regions in which the coherence scalar ψ sits in different minima of $V_{\text{eff}}(\psi)$ (or of a suitably shifted version thereof), and they carry a finite energy per unit area determined by the shape of the potential between the vacua.

In the N–branch framework introduced in the coherence dictionary, the minima of the multi–vacuum potential are labelled by an index $b \in \mathcal{B}$, with field values ψ_b and associated coherence densities $C^{(b)}_{\text{phys}}(x)$. A static domain wall is then more precisely described as an interface between two branches i and j, realised by a profile $\psi_{ij}(z)$ satisfying

$$\lim_{z\to-\infty} \psi_{ij}(z) = \psi_i, \qquad \lim_{z\to+\infty} \psi_{ij}(z) = \psi_j, \tag{21}$$

for some ordered pair (i, j) of branches. The corresponding wall tension is obtained by integrating the excess energy density relative to either vacuum,

$$\sigma_{ij} \;\equiv\; \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\Big(\frac{d\psi_{ij}}{dz}\Big)^2 + V_{\text{eff}}(\psi_{ij}(z)) - V_{\text{eff}}(\psi_i)\right], \tag{22}$$

which reduces to the standard expression once the first–integral relation between $\psi'_{ij}(z)$ and $V_{\text{eff}}(\psi)$ is used. A large–scale domain–wall network can then be viewed as a spatially varying *branch–occupancy field* $b(x) \in \mathcal{B}$ that is piecewise constant in space, with domain walls located on the codimension–one interfaces where $b(x)$ jumps from one value (say i) to another (say j). In this language, the wall tension σ_{ij} and thickness of the corresponding $\psi_{ij}(z)$ profile characterise the cost of changing branch occupancy across the interface.

In the full Taiji–Skyrme–Maxwell theory, the total stress–energy tensor receives contributions from the Skyrme sector, the coherence scalar, the Maxwell field, and any additional matter components. In principle, electromagnetic fields could therefore modify the structure and energetics of coherence walls through $T^{(\text{EM})}_{\mu\nu}$. In the present document we work in an electromagnetically silent regime appropriate for neutral coherence interfaces:

$$A_\mu = 0, \qquad F_{\mu\nu} = 0, \tag{23}$$

so that the wall tension and profile are sourced entirely by the coherence scalar and its coupling to the Skyrme activity. Electromagnetic contributions to the stress–energy tensor are set to zero in this background. Configurations in which coherence walls carry electric charge or magnetic flux would require solving the coupled scalar–Skyrme–Maxwell system with $T^{(\text{EM})}_{\mu\nu} \neq 0$ and time–dependent A_μ; such "charged" or magnetised walls are not treated here and are left for separate work.

To study the interface profiles in the neutral, EM–silent regime, we consider a static, planar wall oriented perpendicular to the z–axis, with ψ depending only on the coordinate z. We also focus on a regime in which the Skyrme activity $K(U)$ varies slowly across the

wall or can be approximated by a constant K_0 on the scale of the wall thickness. Under these assumptions, the equation of motion (**??**) reduces to

$$\frac{d^2\psi}{dz^2} = \frac{dV_{\text{eff}}}{d\psi}(\psi) - \alpha K_0, \tag{24}$$

where we have used the mostly–plus metric signature and α parameterises the strength of the coupling to the Skyrme activity functional.

The constant source term αK_0 can be absorbed into an effective potential

$$\tilde{V}(\psi) = V_{\text{eff}}(\psi) - \alpha K_0 \psi, \tag{25}$$

so that (24) takes the canonical form

$$\frac{d^2\psi}{dz^2} = \frac{d\tilde{V}}{d\psi}(\psi). \tag{26}$$

This admits the standard mechanical analogy: $\psi(z)$ is interpreted as the position of a particle moving in the inverted potential $-\tilde{V}(\psi)$ with "time" z. Static domain–wall solutions correspond to trajectories that start near one minimum of $\tilde{V}$ as $z \to -\infty$ and approach another minimum as $z \to +\infty$, with finite energy per unit area.

Multiplying (26) by $d\psi/dz$ and integrating once with respect to z yields the first integral

$$\frac{1}{2}\left(\frac{d\psi}{dz}\right)^2 = \tilde{V}(\psi) - \tilde{V}(\psi_-), \tag{27}$$

where ψ_- is the value of ψ in the left asymptotic vacuum and the integration constant has been chosen such that the kinetic term vanishes as $z \to -\infty$. For a wall connecting ψ_- to ψ_+, the right–hand side is positive in the intermediate region and vanishes at both endpoints.

The wall profile can then be expressed implicitly as

$$z - z_0 = \pm \int_{\psi_-}^{\psi(z)} \frac{d\psi'}{\sqrt{2\left[\tilde{V}(\psi') - \tilde{V}(\psi_-)\right]}}, \tag{28}$$

with z_0 setting the wall centre. For specific choices of $\tilde{V}$ this integral can sometimes be evaluated in closed form; in general, it provides a well–defined procedure for obtaining the profile numerically. The wall thickness and tension thereby follow from the shape of $\tilde{V}(\psi)$ between the vacua. In regions where K_0 is negligible, $\tilde{V}$ reduces to V_{eff}, and the wall is determined purely by the multi–vacuum potential in the coherence sector.

The existence and properties of domain walls therefore depend on

- the locations and depths of the minima of $\tilde{V}(\psi)$,
- the height and width of the barriers between neighbouring vacua,
- and the effective coupling αK_0 to the Skyrme sector (in the EM–silent regime).

As parameters in $V_{\rm eff}$ and in the interaction term are varied, the positions and even the number of minima can change, leading to bifurcations in the vacuum structure and qualitative changes in the domain–wall network. Within the assumptions (23) the electromagnetic sector does not contribute directly to these walls; it re–enters only through its standard rôle in the total stress–energy tensor when the coherence vacuum and wall configurations are embedded in a larger gravitational or cosmological setting.

6 Bifurcation and Parameter Dependence

The vacuum structure encoded in $V_{\rm eff}(\psi)$ depends on the parameters entering the potential, such as the stiffness λ, the target branch positions ψ_i, and any deformation parameters contained in $\Delta V(\psi)$. It is convenient to collect these into a set of control parameters, schematically denoted by $\boldsymbol{h}$, so that we write

$$V_{\rm eff}(\psi; \boldsymbol{h}). \tag{29}$$

As $\boldsymbol{h}$ varies, the number and nature of minima can change. Bifurcations occur when minima and maxima collide and annihilate, or when new extrema are created.

The conditions for a vacuum at $\psi = \psi_\star$ are

$$V'_{\rm eff}(\psi_\star; \boldsymbol{h}) = 0, \qquad V''_{\rm eff}(\psi_\star; \boldsymbol{h}) > 0. \tag{30}$$

A saddle-node bifurcation, in which a minimum and a neighbouring maximum coalesce and disappear, occurs when

$$V'_{\rm eff}(\psi_\star; \boldsymbol{h}_{\rm crit}) = 0, \qquad V''_{\rm eff}(\psi_\star; \boldsymbol{h}_{\rm crit}) = 0, \tag{31}$$

with a nonvanishing third derivative,

$$V'''_{\rm eff}(\psi_\star; \boldsymbol{h}_{\rm crit}) \neq 0. \tag{32}$$

These equations determine both the critical parameter values $\boldsymbol{h}_{\rm crit}$ and the critical field value $\psi_\star$ at which the bifurcation occurs.

For the polynomial class (13), the conditions (31) translate into algebraic equations in $(\psi_\star, \boldsymbol{h})$. Explicit expressions can be obtained for specific choices of N and of $\Delta V(\psi)$; for example, in a double-well with an external bias parameter h included linearly in ΔV, one recovers the standard fold (saddle-node) structure of a tilted double-well potential. In higher-dimensional parameter spaces, the set of points $(\psi_\star, \boldsymbol{h})$ satisfying (31) can form curves or surfaces along which pairs of extrema coalesce.

In scenarios with three or more branches, the potential may exhibit cusp-like behaviour as two saddle-node lines intersect in control-parameter space. In such cases, one can often map the local form of $V_{\rm eff}(\psi; \boldsymbol{h})$ near the bifurcation point to the canonical cusp potential

$$V_{\rm cusp}(x; a, b) = \frac{1}{4}x^4 + \frac{1}{2}ax^2 + bx, \tag{33}$$

by a suitable rescaling and shift of ψ and of the control parameters. This provides a universal description of the qualitative changes in the number and stability of minima as (a, b) (and thus $\boldsymbol{h}$) are varied.

From the perspective of coherence vacuum structure, the key point is that:

- the number of branches and their positions $\psi_i(\boldsymbol{h})$ can change as parameters vary,
- domain walls may appear or disappear when the corresponding pairs of minima are created or annihilated,
- and the tension and width of walls are sensitive to the barrier heights and curvatures encoded in $V_{\text{eff}}(\psi; \boldsymbol{h})$.

This parameter dependence underlies any large-scale reorganisation of coherence patterns in the Taiji–Skyrme system and provides a controlled language for discussing structural transitions in terms of effective field theory.

7 Structural Gravitational Coupling and Summary

The coherence sector described by the effective Lagrangian (11) contributes to the total stress–energy tensor of the Taiji–Skyrme system. Varying the action with respect to the spacetime metric yields the standard expression for the scalar-field contribution,

$$T^{(\psi)}_{\mu\nu} = \partial_\mu\psi\,\partial_\nu\psi - g_{\mu\nu}\left[\frac{1}{2}\partial_\alpha\psi\,\partial^\alpha\psi - V_{\text{eff}}(\psi) - \alpha\psi K(U)\right], \tag{34}$$

which adds to the Skyrme contribution $T^{(U)}_{\mu\nu}$ to form the total

$$T_{\mu\nu}[U,\psi] = T^{(U)}_{\mu\nu} + T^{(\psi)}_{\mu\nu}. \tag{35}$$

This tensor is the unique source entering the Einstein field equations,

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}[U,\psi], \tag{36}$$

and determines the gravitational response of the system. No independent force law involving gradients of ψ or C_{phys} is introduced; any correlation between coherence gradients and gravitational effects arises through their impact on $T_{\mu\nu}$.

In static or quasi-static configurations, the temporal component $T_{00}[U,\psi]$ defines the effective mass density, and the spatial coherence structure (vacuum branches, domain walls, and their organisation) shapes the resulting gravitational field through the usual curvature equations. The detailed mapping from branch data and domain-wall networks to macroscopic observables (mass–radius relations, effective density profiles, rotation curves, and cosmological behaviour) is handled in separate analyses that build upon the vacuum and interface structure developed here.

Summary

We have constructed a field-theoretic description of coherence vacuum structure and domain walls in the Taiji–Skyrme framework. The key steps were:

- defining the dimensionless coherence field ψ and its physical density representation $C_{\text{phys}} = C_\star\psi$,

- introducing an effective scalar Lagrangian with kinetic term, multi-vacuum potential $V_{\text{eff}}(\psi)$, and coupling to a Skyrme activity functional $K(U)$,
- deriving the equation of motion for ψ from this Lagrangian, thereby replacing ad hoc relations with a systematic effective-field-theory treatment,
- constructing a class of multi-vacuum potentials with tunable minima and analysing their stationary points and curvatures,
- obtaining domain-wall profiles via the standard first integral and expressing the wall tension in terms of the effective potential,
- and describing how changes in control parameters lead to bifurcations in the vacuum structure, including saddle-node and cusp-like scenarios.

The coherence field thus appears as a structural scalar degree of freedom whose vacuum branches and interfaces organise the internal configuration space of the Taiji–Skyrme system. Its influence on gravity is mediated entirely by its contribution to the stress–energy tensor, ensuring compatibility with the general-relativistic coupling used elsewhere in the framework. This vacuum and domain-wall analysis provides the foundation upon which more detailed phenomenological studies of galaxy structure and cosmology, based on coherence branch patterns and their dynamics, can be consistently built.