Unified Parameter Scaling in the Taiji–Skyrme Coherence Framework

Coherence Amplitudes, Vacuum Densities, and Gravitationally Relevant Mass–Radius Relations

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Abstract

The Taiji–Skyrme framework couples an $SU(2)$ field to a coherence structure derived from an underlying Taiji spinor.

The microscopic field content is encoded in the Skyrme Lagrangian, while macroscopic behaviour emerges through effective energy densities, vacuum branches, and gravitational response.

In this document we establish a unified parameter scaling scheme that links the coherence amplitude, its dimensionless normalisation, the associated physical coherence density, and the resulting effective mass–radius relations.

We formulate the role of coherence in terms of a dimensionless field ψ, introduce the corresponding density scale $C_\star$, and relate these quantities to the stress–energy tensor $T_{\mu\nu}[U, \psi]$ that sources gravity.

The resulting set of relations provides a coherent bridge between microphysical parameters $(f, e, v, \alpha_C, C_\star)$, vacuum branch values ψ_i, characteristic soliton scales (M, R), and the effective densities entering the Newtonian limit of general relativity. No additional force mechanism beyond curvature is introduced; coherence influences gravitational phenomena solely through its modulation of energy and pressure distributions.

1 Introduction

The Taiji–Skyrme model describes an $SU(2)$ field $U(x)$ whose dynamics are governed by a Skyrme-type Lagrangian augmented by a coherence structure originating from an underlying Taiji spinor $Z(x)$.

The spinor provides a scalar amplitude that quantifies internal alignment, while the group field U carries the topological and dynamical content.

On large scales, the combined system gives rise to effective energy densities, vacuum branches, solitonic cores, and a nontrivial gravitational response.

A consistent analysis of such a system requires a clear hierarchy of parameters. At the microscopic level, one works with the Skyrme parameters (f, e) and a characteristic coherence amplitude scale v.

At the macroscopic level, one is interested in effective densities, characteristic masses and radii of coherent structures, and the way in which these quantities source the gravitational potential in the weak-field regime.

The purpose of this document is to organise these relationships into a unified parameter framework.

We proceed by:

1. defining the coherence amplitude and its dimensionless normalisation,
2. introducing a physical coherence density with a fixed reference scale $C_\star$,
3. relating these coherence quantities to the energy density $T_{00}[U, \psi]$,
4. constructing controlled mass–radius relations for coherent objects,
5. and exhibiting the parameter map that connects microscopic couplings to macroscopic gravitationally relevant observables.

At all stages, coherence is treated as a structural amplitude that modulates the stress–energy tensor.

The gravitational interaction remains entirely curvature-based, arising from the Einstein field equations with source $T_{\mu\nu}[U, \psi]$.

The unified scaling relations derived here are therefore compatible both with the field-theoretic foundations of the model and with its general-relativistic coupling.

The role of this document is purely organisational: it does not introduce additional dynamics or new force laws, but collects and arranges the parameter dependences that are

already implicit in the Taiji–Skyrme Lagrangian, the coherence normalisation, the vacuum-branch structure, and their general-relativistic coupling.

2 Coherence and Core Parameters

The Taiji spinor

$$Z(x) = \begin{pmatrix} \Psi(x) \\ \Phi(x) \end{pmatrix}, \qquad Z \in \mathbb{C}^2, \tag{1}$$

provides a natural scalar coherence amplitude through its norm:

$$C(x) = |Z(x)|^2 = Z^\dagger(x) Z(x) = |\Psi(x)|^2 + |\Phi(x)|^2. \tag{2}$$

The amplitude $C(x)$ quantifies the local intensity of internal alignment encoded in Z. It is non-negative and invariant under global $SU(2)$ rotations; spatial and temporal variations in $C(x)$ reflect changes in the underlying coherence pattern of the field configuration.

To render this amplitude suitable for comparison across distinct regimes, we introduce a characteristic coherence scale $v > 0$ and define the dimensionless normalised field

$$\psi(x) = \frac{C(x)}{v^2}. \tag{3}$$

The quantity $\psi(x)$ serves as the universal coherence variable used in branch classification, vacuum structure, and effective-medium descriptions. It provides a scale-free measure of how strongly aligned the internal degrees of freedom are relative to the reference scale v.

For later use, we also introduce a fixed reference density scale $C_\star$ with dimension

$$[C_\star] = L^{-4},$$

and define the physical coherence density

$$C_{\mathrm{phys}}(x) = C_\star \, \psi(x). \tag{4}$$

This quantity carries the same dimensions as an energy density and will be used to connect coherence to the effective densities entering gravitational and phenomenological analyses.

The core parameter set of the coherence sector is therefore

$$\{v,\ C_\star,\ \alpha_C\}, \tag{5}$$

where α_C will be introduced below as a proportionality factor that links C_{phys} to the energy density T_{00} in effective descriptions.

3 Energy Density and Effective Mass Density

The microscopic dynamics of the Taiji–Skyrme field are governed by an $SU(2)$ Lagrangian of the general form

$$\mathcal{L}[U] = \mathcal{L}_\sigma[U] + \mathcal{L}_{\text{Skyrme}}[U] + \mathcal{L}_{\text{mass}}[U] + \cdots , \tag{6}$$

where the sigma-model term, Skyrme term, and possible mass terms are constructed from the left-invariant currents

$$L_\mu = U^\dagger \partial_\mu U.$$

The associated symmetric stress–energy tensor $T_{\mu\nu}[U]$ is obtained in the standard way, and the energy density is given by its temporal component,

$$\epsilon(x) = T_{00}[U, \psi]. \tag{7}$$

The explicit dependence of T_{00} on ψ arises through any coherence-dependent contributions included in the effective Lagrangian or in the coarse-grained description of the medium. In particular, the physical coherence density $C_{\text{phys}}(x)$ may enter as part of an effective potential or as a modifier of stiffness and gradient terms, thereby influencing the local energy density.

To express the link between coherence and energetics in a compact form, we introduce an effective mapping

$$C_{\text{phys}}(x) = \alpha_C \, T_{00}(x), \tag{8}$$

where α_C is an effective-field-theory parameter that determines how variations in the energy density are represented in the coherence sector. This relation is to be understood as a coarse-grained correspondence: C_{phys} and T_{00} remain distinct quantities, but in regimes where the medium can be treated as approximately homogeneous on the relevant scales, they are proportional.

In the Newtonian limit of general relativity, the effective mass density sourcing the gravitational potential is given by

$$\rho_{\text{eff}}(x) = T_{00}[U, \psi]. \tag{9}$$

Thus, the chain of influence can be summarised schematically as

$$\psi(x) \;\longrightarrow\; C_{\text{phys}}(x) \;\longrightarrow\; T_{00}[U, \psi] \;\longrightarrow\; \rho_{\text{eff}}(x),$$

which in turn determines the gravitational potential via the Poisson equation

$$\nabla^2 \Phi(x) = 4\pi G \, \rho_{\text{eff}}(x). \tag{10}$$

In this way, the coherence parameters $(v, C_\star, \alpha_C)$ are linked to the gravitationally relevant mass density through the intermediate structure of T_{00}. No additional force law is introduced; all gravitational effects arise from the curvature generated by ρ_{eff}.

4 Vacuum Branch Coherence and Densities

In many regimes of interest the Taiji–Skyrme field approaches quasi-homogeneous configurations on sufficiently large scales. In such regions the normalised coherence field $\psi(x)$ tends to settle near one of a discrete set of branch values

$$\psi(x) \simeq \psi_i, \tag{11}$$

where each ψ_i labels a distinct coherence branch or vacuum sector. The corresponding physical coherence densities are

$$C^{(i)}_{\rm phys} = C_\star\, \psi_i. \tag{12}$$

Given a spatial domain Ω lying predominantly in branch i, we may define the average energy density

$$\bar{\epsilon}_i = \frac{1}{V_\Omega} \int_\Omega d^3x\, T_{00}[U, \psi], \qquad V_\Omega = \int_\Omega d^3x, \tag{13}$$

and the corresponding effective mass density

$$\bar{\rho}_i = \bar{\epsilon}_i. \tag{14}$$

In a coarse-grained description, the pair $(\psi_i, \bar{\rho}_i)$ thus characterises the structural and energetic state of branch i. The coherence parameter ψ_i encodes the internal alignment pattern, while $\bar{\rho}_i$ determines how strongly that branch contributes to the gravitational field through the Einstein equations.

It is often convenient to regard the set

$$\{\psi_i,\ C^{(i)}_{\rm phys},\ \bar{\rho}_i\} \tag{15}$$

as the branch-level parameter triplet summarising the coherence and density properties of each vacuum sector. Transitions between branches, and the interfaces they form, will be governed by the detailed form of the effective potential $V_{\rm eff}(\psi)$ and its coupling to the Skyrme dynamics, but the global scaling relations depend primarily on these averaged quantities.

5 Soliton Core Scaling and Characteristic Radii

Topologically nontrivial configurations of the $SU(2)$ field $U(x)$ give rise to solitonic objects whose energy is localised within a finite spatial region. Such configurations typically interpolate between different coherence branches or approach a common branch value at spatial infinity while exhibiting a nontrivial structure in the interior.

Let us consider an approximately spherically symmetric soliton centred at the origin, with a characteristic radius R, beyond which the fields relax toward a branch value ψ_∞ and the energy density decays rapidly. We define the *core region* as the ball

$$\mathcal{B}_R = \{\boldsymbol{x} \in \mathbb{R}^3 \,|\, |\boldsymbol{x}| \le R\}, \tag{16}$$

inside which the coherence field and energy density are significantly elevated relative to the asymptotic background.

The total soliton mass is

$$M = \int d^3x\, T_{00}[U,\psi] = 4\pi \int_0^\infty dr\, r^2\, \epsilon(r), \tag{17}$$

with $\epsilon(r)$ the spherically averaged energy density.

We introduce the *core-averaged density*

$$\bar{\rho}_{\rm core} = \frac{1}{V_R}\int_{\mathcal{B}_R} d^3x\, T_{00}[U,\psi], \qquad V_R = \frac{4\pi}{3}R^3, \tag{18}$$

which provides an effective uniform density for the interior region. In terms of $\bar{\rho}_{\rm core}$, the soliton mass can be written as

$$M = \bar{\rho}_{\rm core} V_R\, S(\mu), \tag{19}$$

where $S(\mu)$ is a dimensionless structural factor that accounts for:

- deviations of $\epsilon(r)$ from strict uniformity inside R,
- contributions from the tail region $r > R$,
- and dependence on the dimensionless Skyrme parameters collectively denoted by μ.

The factor $S(\mu)$ is obtained from the dimensionless profile functions solving the Skyrme equations for given parameter values. By construction, $S(\mu)$ is of order unity for well-localised solitons, but its precise value encodes the detailed internal structure of the configuration.

The link to coherence is provided by the core-averaged physical coherence density,

$$\bar{C}_{\rm phys,core} = \frac{1}{V_R}\int_{\mathcal{B}_R} d^3x\, C_{\rm phys}(x) = C_\star\, \bar{\psi}_{\rm core}, \tag{20}$$

with

$$\bar{\psi}_{\rm core} = \frac{1}{V_R}\int_{\mathcal{B}_R} d^3x\, \psi(x). \tag{21}$$

The pair $(\bar{\rho}_{\rm core}, \bar{\psi}_{\rm core})$ therefore summarises the energetic and coherence structure of the soliton interior.

6 Mass–Radius Relations

The mass–radius relation for a solitonic object in the Taiji–Skyrme coherence framework follows from Eq. (19). In terms of the core-averaged density, we have

$$M(R) = \frac{4\pi}{3}\,\bar{\rho}_{\rm core}\, R^3\, S(\mu). \tag{22}$$

This expression exhibits the expected cubic scaling with radius for a quasi-uniform core, modulated by the structural factor $S(\mu)$. The latter encodes how the detailed Skyrme profile, coherence distribution, and tail behaviour modify the naive uniform-density estimate.

From the perspective of parameter unification, it is useful to make explicit the dependence of $\bar{\rho}_{\rm core}$ on the microscopic parameters. At the level of dimensional analysis, the Skyrme scale f, the dimensionless coupling e, and the coherence scale $C_\star$ combine to set the characteristic magnitude of T_{00} and hence of $\bar{\rho}_{\rm core}$. We may therefore write schematically

$$\bar{\rho}_{\rm core} \sim f^4\, \mathcal{F}(e, \bar{\psi}_{\rm core}, \dots), \tag{23}$$

where $\mathcal{F}$ is a dimensionless function determined by the specific form of the Lagrangian and the coherence-dependent terms.

Inserting this into the mass–radius relation yields

$$M(R) \sim \frac{4\pi}{3} f^4\, \mathcal{F}(e, \bar{\psi}_{\rm core}, \dots)\, R^3\, S(\mu). \tag{24}$$

This relation identifies the combination of parameters that control the macroscopic properties of the soliton:

- f sets the overall energy scale,
- e influences the stiffness and the profile shape through $\mathcal{F}$ and μ,
- $\bar{\psi}_{\rm core}$ encodes the internal coherence level,
- $S(\mu)$ captures structural corrections.

The resulting mass–radius scaling is compatible with general relativity as long as R is sufficiently small that the weak-field approximation holds and the soliton does not become gravitationally self-trapped. In more extreme regimes, one would need to solve the coupled Einstein–Skyrme–coherence system to obtain fully relativistic mass–radius curves, but the parameter dependences identified here remain relevant.

7 Unified Parameter Map and Summary

The analysis above can be summarised as a parameter map connecting microscopic field-theoretic quantities to macroscopic gravitationally relevant observables.

At the microscopic level, the primary parameters are

$$\{f,\ e,\ v,\ C_\star,\ \alpha_C\}, \tag{25}$$

together with the detailed form of the effective potential $V_{\rm eff}(\psi)$ and any coherence-dependent modifications of the Skyrme sector.

From these, one constructs:

- the coherence amplitude $C = |Z|^2$,
- the dimensionless field $\psi = C/v^2$,
- the physical coherence density $C_{\rm phys} = C_\star \psi$,

- the energy density $T_{00}[U, \psi]$,
- the effective mass density $\rho_{\text{eff}} = T_{00}$.

Vacuum branches are characterised by

$$\{\psi_i,\ C^{(i)}_{\text{phys}},\ \bar{\rho}_i\}, \tag{26}$$

providing a coherent description of the structural and energetic properties of each sector. Solitonic configurations are then described by:

$$\{\bar{\psi}_{\text{core}},\ \bar{\rho}_{\text{core}},\ R,\ M,\ S(\mu)\}, \tag{27}$$

where R is a characteristic radius and M the total mass.

The mapping

$$(f, e, v, C_\star, \alpha_C;\ \psi_i, V_{\text{eff}}) \ \longrightarrow\ (\bar{\rho}_i, \bar{\rho}_{\text{core}}, R, M, \Phi)$$

summarises how the microscopic parameters and coherence structure determine the macroscopic gravitational response. At no stage is an additional force law introduced; the gravitational potential Φ is entirely determined by the effective mass density via the Poisson equation or, in the fully relativistic case, by the Einstein field equations.

In this way, the unified parameter scaling framework presented here provides a consistent bridge between:

- the Taiji–Skyrme Lagrangian and coherence sector,
- the classification of vacuum branches,
- the structure and scaling of solitonic objects,
- and the gravitational fields they source.

This establishes a coherent foundation upon which more detailed phenomenological modelling and numerical investigations can be built.

Illustrative Parameter Mapping

To make the parameter flow more concrete, it is useful to display a schematic example in which each step of the mapping is carried out explicitly. Consider a regime in which the microscopic parameters have been fixed at some reference values

$$f = f_0, \qquad e = e_0, \qquad v = v_0, \qquad C_\star = C_{\star,0}, \qquad \alpha_C = \alpha_{C,0},$$

and suppose that the effective potential $V_{\text{eff}}(\psi)$ possesses a vacuum branch with normalised coherence

$$\psi = \psi_{\text{core}},$$

which dominates inside a given solitonic core.

From the coherence sector we obtain the corresponding physical coherence density in the core,

$$C_{\mathrm{phys,core}} = C_{\star,0}\,\psi_{\mathrm{core}}. \tag{28}$$

Through the effective mapping (8), this is related to the core energy density by

$$C_{\mathrm{phys,core}} = \alpha_{C,0}\,\bar{\rho}_{\mathrm{core}}, \tag{29}$$

so that

$$\bar{\rho}_{\mathrm{core}} = \frac{C_{\star,0}}{\alpha_{C,0}}\,\psi_{\mathrm{core}}. \tag{30}$$

On the other hand, the Skyrme sector determines the dimensionless structural factor $S(\mu_0)$ for the chosen values (f_0, e_0), with μ_0 denoting the corresponding dimensionless parameter set.

For a soliton with characteristic radius R, the mass–radius relation

$$M(R) = \frac{4\pi}{3}\,\bar{\rho}_{\mathrm{core}}\,R^3\,S(\mu_0) \tag{31}$$

then yields a core mass

$$M(R) = \frac{4\pi}{3}\,\frac{C_{\star,0}}{\alpha_{C,0}}\,\psi_{\mathrm{core}}\,R^3\,S(\mu_0). \tag{32}$$

Thus, for fixed microscopic parameters and a given branch coherence ψ_{core}, the mass and radius of the soliton are linked through the combination

$$\left(\frac{C_{\star,0}}{\alpha_{C,0}}\,\psi_{\mathrm{core}}\,S(\mu_0)\right),$$

which encapsulates the influence of coherence and Skyrme structure on the macroscopic properties of the object.

In a gravitational setting, the effective mass density ρ_{eff} entering the Poisson equation is determined by the same core density $\bar{\rho}_{\mathrm{core}}$ together with the contribution of the surrounding medium. The example above therefore illustrates explicitly how a choice of microscopic Taiji–Skyrme and coherence parameters propagates through to the effective densities and mass–radius relations that enter macroscopic and gravitational modelling.

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8 Numerical Interface and Electromagnetic Extensibility

The illustrative mapping above shows explicitly how a choice of microscopic Taiji–Skyrme and coherence parameters propagates through to core densities, mass–radius relations, and the effective quantities that enter macroscopic and gravitational modelling. In practice, these relations are evaluated and tested within numerical solvers that evolve the Skyrme–coherence system in time. The unified scaling constructed in this note is therefore also the natural interface between the field theory and the dynamical simulation infrastructure.

In the present implementation, numerical experiments evolve the classical Skyrme and coherence fields only. The dynamical variables are

$$U(x,t), \qquad \psi(x,t),$$

with initial data chosen on a given coherence branch and boundary conditions fixed by the physical scenario (isolated soliton, core–halo configuration, etc.). At each timestep the solver reconstructs

- the local energy density $T_{00}[U,\psi]$,
- the effective mass density $\rho_{\text{eff}}(x,t) = T_{00}[U,\psi]$ in the EM–silent regimes of interest,
- and integrated quantities such as the core–averaged density $\bar{\rho}_{\text{core}}$, total mass M, and characteristic radius R,

using the same parameter combinations and structural factor $S(\mu)$ that appear in the mass–radius analysis. In weak–field settings, the gravitational potential is then obtained from ρ_{eff} via the Newtonian limit of general relativity, in accordance with the Maxwell–extended normalisation and gravity documents.

At the level of the full Taiji–Skyrme–Maxwell theory, the internal U(1) coherence connection A_μ with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ contributes to the total stress–energy tensor through the standard Maxwell term $T^{(\text{EM})}_{\mu\nu}$. The effective density entering the Poisson or Einstein equations can be written schematically as

$$\rho_{\text{eff}}(x) = T^{\text{tot}}_{00}(x) = T^{(\text{baryon})}_{00}(x) + T^{(\text{coh})}_{00}(x) + T^{(\text{EM})}_{00}(x) + T^{(\text{other})}_{00}(x).$$

The numerical studies targeted by the present scaling, however, concern regimes in which the electromagnetic contribution to the bulk mass–radius relations is negligible:

$$T^{(\text{EM})}_{00}(x) \ll T^{(\text{coh})}_{00}(x) + T^{(\text{baryon})}_{00}(x),$$

so that, operationally,

$$\rho_{\text{eff}}(x) \simeq T^{(\text{coh})}_{00}(x) + T^{(\text{baryon})}_{00}(x).$$

This approximation is implemented in the solver by working in an electromagnetically silent background,

$$A_\mu(x,t) = 0, \qquad F_{\mu\nu}(x,t) = 0,$$

and by omitting $T^{(\text{EM})}_{00}$ from the diagnostic reconstruction. No additional force law is introduced: gravity remains entirely curvature–based and is sourced by ρ_{eff} in the standard way.

Although current simulations are EM–silent, the unified parameter structure has been chosen so that electromagnetic degrees of freedom can be incorporated without redesigning the numerical architecture. An EM–extended evolution would introduce, in addition to U and ψ, either

$$A_i(x,t) \quad \text{or} \quad (E_i(x,t), B_i(x,t)),$$

as dynamical variables defined on the same spatial grid. Maxwell's equations,

$$\nabla_\nu F^{\nu\mu} = g_{\mathrm{EM}}^2 J^\mu[\tilde{Z}],$$

with J^μ constructed from the covariant derivative of the Taiji spinor $\tilde{Z}$, would then be evolved alongside the Skyrme and coherence equations of motion. The Hamiltonian density would acquire an electromagnetic contribution of the schematic form

$$\mathcal{H}_{\mathrm{EM}} \propto E^2 + B^2,$$

and the diagnostics would be extended to include electromagnetic energy, Poynting flux, and gauge–invariant field invariants, in addition to the coherence and Skyrme observables already tracked.

From a software–design perspective, field components, boundary conditions, and output channels in the current solver are organised so that additional variables such as A_i or (E_i, B_i) can be appended rather than requiring a structural rewrite. The role of this section is therefore to make explicit that:

- the unified parameter scaling developed in this document underlies both the analytical mass–radius relations and the present EM–silent simulations;
- the same framework is ready to accommodate a future extension in which electromagnetic energy density and stresses are treated on the same footing as coherence and Skyrme contributions in both the dynamics and the gravitational sector.