Coherence Normalisation in the Taiji–Skyrme Framework

Dimensional Structure, Natural Scales, and Effective Gravitational Source Mapping

Eduardo Santibañez

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Abstract

This document establishes a consistent dimensional normalisation for the coherence field in the Taiji–Skyrme framework, in a form compatible with the unified coherence dictionary and a purely GR-based gravitational sector. Starting from the SU(2) chiral Lagrangian with Skyrme stabilisation, we derive the dimensions of the fundamental parameters (f, e, m) and recall the Taiji spinor amplitude that underlies the classical coherence order parameter. We then construct a physical coherence density $C_{\rm phys}(x)$ with the dimensions of an energy density and introduce an effective identification $C_{\rm phys}(x) \simeq \alpha_C\, \varepsilon_{\rm coh}(x)$ with the local *coherence-sector* energy density $\varepsilon_{\rm coh}(x) = T^{(\rm coh)}_{00}(x)$, explicitly labelled as a phenomenological mapping and not as a fundamental definition.

Using the scaling properties of Skyrmion solutions, we derive a natural coherence scale $C_0 \propto e^2 f^4$ and introduce a dimensionless coherence field $\tilde{C}(x) = C_{\rm phys}(x)/C_0$. We express the global coherence charge in terms of $\tilde{C}$ and show that, for a single soliton, it scales consistently with soliton mass and radius. Finally, we discuss how the normalised coherence field enters gravitational physics only through its contribution to the total stress–energy tensor $T^{\rm tot}_{\mu\nu}$ in the Einstein equations. The total energy density $\rho_{\rm eff} = T^{\rm tot}_{00}$ is decomposed as $\rho_{\rm eff} = \rho_{\rm baryon} + \rho_{\rm coh} + \rho_{\rm EM} + \rho_{\rm other}$, where $\rho_{\rm EM} \equiv T^{(\rm EM)}_{00}$ is the electromagnetic contribution from the Maxwell sector. Any local relation of the type $a(x) \propto \nabla\tilde{C}(x)$ is strictly interpreted as a phenomenological, Newtonian-limit parametrisation of coherence-induced mass distributions, not as an independent fundamental force law.

All subsequent uses of coherence—in emergent macroscopic phenomenology, quantum coherence operators, and unified particle scaling—can therefore be expressed in terms of the dimensionless field $\tilde{C}$ and explicit parameter-dependent prefactors built from (f, e, m) and dimensionless constants such as α_C and η_C, without introducing a second gravitational mechanism or omitting the electromagnetic contribution to the total energy density.

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1 Skyrme–Taiji Core and Dimensional Structure

1.1 Field content and Lagrangian

We consider an SU(2)-valued field

$$U(x) \in SU(2), \qquad U^\dagger U = \mathbb{1}, \qquad \det U = 1, \tag{1}$$

governed by the standard chiral Lagrangian with Skyrme stabilisation and a mass term,

$$L = \frac{f^2}{4}\,\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\,\mathrm{Tr}\big([U^\dagger\partial_\mu U,\, U^\dagger \partial_\nu U]^2\big) - V(U). \tag{2}$$

Here

- f is a characteristic energy scale (Taiji stiffness),
- e is a dimensionless coupling constant,
- $V(U)$ contains the mass term and possible small anisotropies, typically of the form $V(U) \sim m^2 f^2\,\mathrm{Tr}(\mathbb{1} - U)$.

We work in natural units with $\hbar = c = 1$ unless explicitly stated otherwise.

The Taiji spinor $Z(x)$ appears in the broader framework as a two-component complex field whose classical norm $|Z(x)|^2$ encodes the local coherence amplitude. In this note we only need that Z is dimensionless at the classical level:

$$Z(x) = \begin{pmatrix} \Psi(x) \\ \Phi(x) \end{pmatrix}, \qquad C_{\mathrm{cl}}(x) \equiv |Z(x)|^2 = |\Psi(x)|^2 + |\Phi(x)|^2, \qquad [Z] = [C_{\mathrm{cl}}] = 1. \tag{3}$$

1.2 Dimensional analysis

The action reads

$$S = \int d^4x\, L. \tag{4}$$

In natural units, the action is dimensionless, so the Lagrangian density has

$$[L] = L^{-4}, \tag{5}$$

where L denotes length. The field $U(x)$ is SU(2)-valued and therefore

$$[U] = 1 \quad \text{(dimensionless)}. \tag{6}$$

Space–time derivatives have

$$[\partial_\mu] = L^{-1}. \tag{7}$$

For the kinetic term,

$$L_2 = \frac{f^2}{4}\,\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U), \tag{8}$$

we have

$$[\partial_\mu U] = L^{-1}, \qquad \big[\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)\big] = L^{-2}. \tag{9}$$

Thus

$$[L_2] = [f^2]\,L^{-2} = L^{-4} \quad \Rightarrow \quad [f^2] = L^{-2}, \qquad [f] = L^{-1}. \tag{10}$$

For the Skyrme term,

$$L_4 = \frac{1}{32e^2}\mathrm{Tr}\big([U^\dagger\partial_\mu U,\, U^\dagger\partial_\nu U]^2\big)\,, \tag{11}$$

each factor $U^\dagger\partial_\mu U$ has dimension L^{-1}, hence the commutator is L^{-1} and the squared commutator is L^{-2}:

$$\big[\mathrm{Tr}\big([U^\dagger\partial_\mu U,\, U^\dagger\partial_\nu U]^2\big)\big] = L^{-4}. \tag{12}$$

It follows that

$$[L_4] = [e^{-2}]\, L^{-4} = L^{-4} \quad \Rightarrow \quad [e] = 1. \tag{13}$$

A mass term of the form

$$V(U) \sim m^2 f^2\, \mathrm{Tr}(\mathbb{1} - U) \tag{14}$$

requires

$$[m^2 f^2] = L^{-4} \quad \Rightarrow \quad [m^2] = L^{-2}, \qquad [m] = L^{-1}. \tag{15}$$

Summarising:

$$[f] = L^{-1}, \qquad [e] = 1, \qquad [m] = L^{-1}, \qquad [L] = L^{-4}. \tag{16}$$

2 Coherence Density and Effective Mapping to Energy Density

2.1 Classical coherence order parameter

In the unified coherence dictionary, the classical coherence order parameter is constructed from the Taiji spinor norm. We introduce a dimensionless order parameter $\psi(x)$ via

$$\psi(x) = \frac{C_{\mathrm{cl}}(x)}{v^2} = \frac{|Z(x)|^2}{v^2}, \tag{17}$$

where v is a fixed dimensionless normalisation constant.[1] By construction,

$$[\psi] = 1. \tag{18}$$

To interface with gravity and macroscopic phenomenology we require a field with the dimensions of an energy density. We therefore define a physical coherence density $C_{\mathrm{phys}}(x)$ by

$$C_{\mathrm{phys}}(x) = C_\star\, \psi(x), \tag{19}$$

where $C_\star$ is a fixed constant with the same dimensions as the energy density:

$$[C_\star] = L^{-4}, \qquad [C_{\mathrm{phys}}] = L^{-4}. \tag{20}$$

The choice of $C_\star$ is model-dependent and will be related to the Skyrmion core energy density in Sec. 3.

[1] In the Taiji–Skyrme dictionary, v is chosen such that typical coherent configurations satisfy $\psi \sim \mathcal{O}(1)$ in the interior region.

2.2 Effective identification with local energy density

Let $\varepsilon_{\mathrm{coh}}(x)$ denote the *coherence-sector* contribution to the local energy density derived from the Skyrme–Taiji Lagrangian, i.e. the 00-component of the stress–energy tensor associated with the U field and its coherence dressing:

$$\varepsilon_{\mathrm{coh}}(x) \equiv T_{00}^{(\mathrm{coh})}(x), \qquad [\varepsilon_{\mathrm{coh}}] = L^{-4}. \tag{21}$$

This quantity does not include the electromagnetic or purely baryonic components, which are accounted for separately at the level of the total stress–energy tensor.

In regimes where the coherence order parameter is tightly correlated with the underlying Skyrmion/coherence energy density, it is convenient to identify the physical coherence density with $\varepsilon_{\mathrm{coh}}$ up to a dimensionless factor:

$$C_{\mathrm{phys}}(x) \simeq \alpha_C \, \varepsilon_{\mathrm{coh}}(x), \tag{22}$$

where

$$\alpha_C \text{ is dimensionless.} \tag{23}$$

This relation is explicitly not a fundamental definition of coherence. Rather, it is an effective identification valid in calibrated regimes, making precise the statement that, on coarse-grained scales, the coherence field tracks the Skyrmion/Coherence-sector contribution to the energy density.

Combining (19) and (22) yields

$$C_\star \, \psi(x) \simeq \alpha_C \, T_{00}^{(\mathrm{coh})}(x). \tag{24}$$

Given $C_\star$ and α_C, this determines the typical magnitude of ψ in a given configuration, without conflating the coherence calibration with electromagnetic or baryonic contributions.

3 Natural Coherence Scale

3.1 Skyrmion scaling and core energy density

Static Skyrmion solutions are characterised by an energy M and a core radius R. In the standard Skyrme model, one can rescale spatial coordinates via

$$r = \frac{\rho}{ef}, \tag{25}$$

which implies that the characteristic size of the soliton scales as

$$R \sim \frac{1}{ef}. \tag{26}$$

The total soliton mass scales as

$$M \sim \frac{f}{e}, \tag{27}$$

up to a dimensionless profile-dependent factor of order unity.

A natural estimate for the core energy density is therefore

$$\varepsilon_{\mathrm{core}} \sim \frac{M}{R^3} \sim \frac{f/e}{(1/(ef))^3} \sim e^2 f^4, \tag{28}$$

again up to a dimensionless factor that depends on the specific hedgehog solution.

3.2 Core energy density and natural coherence scale

Motivated by the estimate above, we define a natural coherence scale C_0 by

$$C_0 = \alpha_C e^2 f^4, \tag{29}$$

with α_C the same dimensionless parameter that appears in (22). By construction,

$$[C_0] = L^{-4}. \tag{30}$$

The choice of α_C fixes the relative normalisation between the coherence density and the core energy density of Skyrmions. For typical soliton solutions, C_0 is of the same order of magnitude as $\varepsilon_{\text{core}}$.

4 Dimensionless Coherence Field

4.1 Definition

We now introduce a dimensionless, normalised coherence field $\tilde{C}(x)$ by

$$\tilde{C}(x) = \frac{C_{\text{phys}}(x)}{C_0} = \frac{C_\star}{C_0}\,\psi(x), \tag{31}$$

with C_0 given by (29). The dimensions satisfy

$$[\tilde{C}] = 1, \tag{32}$$

as expected for a dimensionless field.

In the regime where the effective identification (22) holds, (31) becomes

$$\tilde{C}(x) \simeq \frac{\alpha_C}{C_0}\, T_{00}^{(\text{coh})}(x) = \frac{T_{00}^{(\text{coh})}(x)}{e^2 f^4}, \tag{33}$$

up to a dimensionless factor. Thus $\tilde{C}$ can be regarded as the local *coherence-sector* energy density expressed in units of the Skyrmion core scale $e^2 f^4$.

4.2 Global coherence charge in normalised form

We define the global coherence charge Q_C as the spatial integral of the physical coherence density,

$$Q_C \equiv \int d^3x\, C_{\text{phys}}(x). \tag{34}$$

Using (31) we can rewrite this as

$$Q_C = \int d^3x\, C_0 \tilde{C}(x) = C_0 \int d^3x\, \tilde{C}(x). \tag{35}$$

For a single, approximately spherical Skyrmion of radius $R \sim 1/(ef)$ and core density of order C_0, we have

$$Q_C \sim C_0 R^3 \sim (e^2 f^4)\,\frac{1}{(ef)^3} \sim \frac{f}{e}, \tag{36}$$

so that Q_C scales with the soliton mass $M \sim f/e$, up to dimensionless profile factors. Thus the global coherence charge is consistent with the known Skyrmion mass–radius scaling.

5 Coherence and Gravity with Normalised Coherence

In the Taiji–Skyrme framework, gravity is described by general relativity. The metric $g_{\mu\nu}$ satisfies the Einstein equations

$$G_{\mu\nu} = 8\pi G\, T^{\rm tot}_{\mu\nu}(x), \tag{37}$$

where $T^{\rm tot}_{\mu\nu}$ is the *total* stress–energy tensor. It can be decomposed schematically as

$$T^{\rm tot}_{\mu\nu} = T^{(\rm baryon)}_{\mu\nu} + T^{(\rm coh)}_{\mu\nu} + T^{(\rm EM)}_{\mu\nu} + T^{(\rm other)}_{\mu\nu}, \tag{38}$$

with:

- $T^{(\rm baryon)}_{\mu\nu}$ the stress–energy of ordinary matter (baryons, gas, stars, etc.),
- $T^{(\rm coh)}_{\mu\nu}$ the Skyrme–Taiji coherence-sector contribution,
- $T^{(\rm EM)}_{\mu\nu}$ the electromagnetic stress–energy tensor associated with the Maxwell sector and the coherence connection,
- $T^{(\rm other)}_{\mu\nu}$ any additional matter or dark components.

The coherence field influences gravity only through its contribution to $T^{(\rm coh)}_{\mu\nu}$ and hence to $T^{\rm tot}_{\mu\nu}$; the Maxwell sector contributes through $T^{(\rm EM)}_{\mu\nu}$.

In particular, in the weak-field, non-relativistic limit one recovers the Newtonian potential Φ via

$$\nabla^2\Phi(x) = 4\pi G\, \rho_{\rm eff}(x), \qquad \rho_{\rm eff}(x) \equiv T^{\rm tot}_{00}(x), \tag{39}$$

and test-body accelerations are given by

$$\mathbf{a}(x) = -\nabla\Phi(x). \tag{40}$$

No additional fundamental "coherence force" is introduced.

It is often convenient to decompose the effective mass density as

$$\rho_{\rm eff}(x) = \rho_{\rm baryon}(x) + \rho_{\rm coh}(x) + \rho_{\rm EM}(x) + \rho_{\rm other}(x), \tag{41}$$

where

$$\rho_{\rm baryon} \equiv T^{(\rm baryon)}_{00}, \quad \rho_{\rm coh} \equiv T^{(\rm coh)}_{00}, \quad \rho_{\rm EM} \equiv T^{(\rm EM)}_{00}, \quad \rho_{\rm other} \equiv T^{(\rm other)}_{00}. \tag{42}$$

In cosmological applications, $\rho_{\rm EM}$ is part of the radiation component with equation of state $w = 1/3$. In many non-relativistic, galaxy-scale applications, $\rho_{\rm EM}$ is much smaller than $\rho_{\rm baryon}$ and $\rho_{\rm coh}$, and can be neglected when computing Φ; this approximation is always to be stated explicitly.

5.1 Branch-resolved coherence contribution

In the multi-branch Taiji coherence framework, the coherence sector of the stress–energy tensor is written as a sum over branches,

$$T_{\mu\nu}^{(\mathrm{coh})}(x) = \sum_{b=1}^{N} T_{\mu\nu}^{(\mathrm{coh},b)}(x), \tag{43}$$

so that the coherence contribution to the energy density decomposes as

$$\rho_{\mathrm{coh}}(x) = T_{00}^{(\mathrm{coh})}(x) = \sum_{b=1}^{N} \rho_{\mathrm{coh},b}(x), \qquad \rho_{\mathrm{coh},b}(x) \equiv T_{00}^{(\mathrm{coh},b)}(x). \tag{44}$$

Equivalently, the effective mass density in (41) may be written as

$$\rho_{\mathrm{eff}}(x) = \rho_{\mathrm{baryon}}(x) + \sum_{b=1}^{N} \rho_{\mathrm{coh},b}(x) + \rho_{\mathrm{EM}}(x) + \rho_{\mathrm{other}}(x), \tag{45}$$

with the understanding that only a small subset of branches contributes appreciably to the sum in a given physical regime (for example, on galaxy scales or in specific cosmological epochs).

At the level of the normalised coherence field, the physical coherence density (19) is likewise regarded as a sum over branch-resolved pieces,

$$C_{\mathrm{phys}}(x) = \sum_{b=1}^{N} C_{\mathrm{phys}}^{(b)}(x), \qquad C_{\mathrm{phys}}^{(b)}(x) = C_\star\, \psi_b(x), \tag{46}$$

where the $\psi_b(x)$ are branch-resolved order parameters satisfying $\sum_b \psi_b(x) = \psi(x)$. In calibrated regimes where the effective mapping (22) holds for each branch separately with the same coefficient α_C, one has

$$C_{\mathrm{phys}}^{(b)}(x) \simeq \alpha_C\, \rho_{\mathrm{coh},b}(x), \tag{47}$$

so that (46) and (44) reproduce the single-sector relation (22). The branch decomposition is therefore a partition of the same C_{phys} and ρ_{coh}, not a second definition.

5.2 Phenomenological force-law structure

For some phenomenological applications (e.g. coarse-grained galaxy dynamics) it is convenient to parametrise the influence of spatially varying coherence on effective accelerations in a reduced, Newtonian description. A common ansatz is to relate an effective acceleration field $\mathbf{a}_{\mathrm{coh}}$ to the gradient of the normalised coherence field $\tilde{C}$,

$$\mathbf{a}_{\mathrm{coh}}(x) = \gamma_C\, \nabla \tilde{C}(x), \tag{48}$$

where γ_C is a phenomenological coefficient.

This relation is not a new fundamental law of gravity. It is a parametrisation of how, in a given effective model, gradients of the coherence field—which in turn track the distribution of coherence-sector energy density and branch structure—might correlate with extra acceleration components in a Newtonian fit. The underlying gravitational dynamics remain entirely encoded in (37) and the decomposition (38). Electromagnetic effects enter only through ρ_{EM} and associated stresses in $T_{\mu\nu}^{(\mathrm{EM})}$, not through $\tilde{C}$ itself.

5.3 Dimensional consistency

Let us determine the dimensions of γ_C in the ansatz (48). In standard Newtonian units,

$$[\mathbf{a}] = L/T^2, \qquad [\nabla] = L^{-1}, \qquad [\tilde{C}] = 1, \tag{49}$$

so necessarily

$$[\gamma_C] = L^2/T^2. \tag{50}$$

The coefficient γ_C may be expressed as

$$\gamma_C = \eta_C \, C_0, \tag{51}$$

with

$$C_0 = \alpha_C e^2 f^4 \tag{52}$$

as in (29). Since $[C_0] = L^{-4}$, the parameter η_C must carry the remaining dimensions such that $[\gamma_C] = L^2/T^2$ is satisfied. It is therefore understood that η_C is a phenomenological quantity whose dimensions encode the coarse-graining scale and the effective time-scale of the reduced Newtonian model.

The upshot is that any ansatz of the form (48) can be made dimensionally consistent using the natural coherence scale C_0, but the resulting γ_C is a model-dependent, emergent coefficient rather than a new universal constant of nature.

5.4 Relation to the dimensionless coherence field

Using (31), the phenomenological ansatz (48) can also be written as

$$\mathbf{a}_{\rm coh}(x) = \gamma_C \, \nabla \left(\frac{C_{\rm phys}(x)}{C_0} \right) = \frac{\gamma_C}{C_0} \, \nabla C_{\rm phys}(x). \tag{53}$$

In regimes where $C_{\rm phys} \simeq \alpha_C T_{00}^{(\rm coh)}$, this further reduces to

$$\mathbf{a}_{\rm coh}(x) \simeq \frac{\gamma_C \alpha_C}{C_0} \, \nabla T_{00}^{(\rm coh)}(x) = \eta_C \, \nabla T_{00}^{(\rm coh)}(x). \tag{54}$$

Again, this is to be interpreted as a coarse-grained parametrisation: regions with strong spatial variation in the coherence-sector energy density can be modelled as contributing an additional acceleration field proportional to $\nabla T_{00}^{(\rm coh)}$, but the fundamental gravitational dynamics remain governed by (37), with $\rho_{\rm eff}$ given by (41) and electromagnetic contributions entering via $\rho_{\rm EM}$ where appropriate.

Electromagnetic contribution and regime dependence

The electromagnetic energy density is defined as

$$\rho_{\rm EM}(x) = T_{00}^{(\rm EM)}(x), \tag{55}$$

where $T_{\mu\nu}^{(\rm EM)}$ is the Maxwell stress–energy tensor of the coherence connection sector in the extended Skyrme–Taiji Lagrangian. In the present normalisation framework, $\rho_{\rm EM}$ is included on the same footing as the other components in the total effective density $\rho_{\rm eff}$ in (41).

In many non-relativistic, galaxy-scale applications, typical magnetic field strengths and plasma conditions imply that $\rho_{\rm EM}$ is several orders of magnitude smaller than both the baryonic mass density $\rho_{\rm baryon}$ and the coherence-structured contribution $\rho_{\rm coh}$. In those regimes it is therefore consistent to set

$$\rho_{\rm EM} \approx 0 \tag{56}$$

for the purposes of computing the gravitational potential, as is done explicitly in the galaxy-rotation phenomenology.

By contrast, in early-universe cosmology the electromagnetic sector forms part of the radiation content with equation of state $w = 1/3$, and its contribution to the total energy density cannot be neglected. In that context, $\rho_{\rm EM}$ is naturally grouped into the radiation component $\rho_r(a)$ appearing in the Friedmann equations.

6 Summary and Outlook

We summarise the main results:

1. The Skyrme–Taiji Lagrangian defines the basic parameters (f, e, m) with dimensions

$$[f] = L^{-1}, \qquad [e] = 1, \qquad [m] = L^{-1}, \qquad [L] = L^{-4}.$$

2. The Taiji spinor $Z(x)$ provides a classical coherence amplitude $C_{\rm cl}(x) = |Z(x)|^2$, from which a dimensionless order parameter $\psi(x) = C_{\rm cl}(x)/v^2$ is constructed.

3. A physical coherence density $C_{\rm phys}(x) = C_\star \psi(x)$ with units of energy density is defined. In calibrated regimes, it can be identified with the *coherence-sector* local energy density via $C_{\rm phys}(x) \simeq \alpha_C T^{(\rm coh)}_{00}(x)$, without conflating it with electromagnetic or purely baryonic contributions.

4. The natural coherence scale is set by the Skyrmion core energy density, $C_0 = \alpha_C e^2 f^4$, providing a reference scale for coherent configurations.

5. A dimensionless normalised coherence field $\tilde{C}(x) = C_{\rm phys}(x)/C_0$ is introduced. The global coherence charge $Q_C = \int d^3x\, C_{\rm phys}(x) = C_0 \int d^3x\, \tilde{C}(x)$ scales with the Skyrmion mass and is consistent with standard mass–radius relations.

6. Gravity is described entirely by general relativity with source $T^{\rm tot}_{\mu\nu}$. Coherence influences gravitational phenomena only via its contribution to $T^{(\rm coh)}_{\mu\nu}$ inside $T^{\rm tot}_{\mu\nu}$. The total effective source density is $\rho_{\rm eff} = \rho_{\rm baryon} + \rho_{\rm coh} + \rho_{\rm EM} + \rho_{\rm other}$, with $\rho_{\rm EM}$ coming from the Maxwell sector. Any relation of the type $\mathbf{a}_{\rm coh} = \gamma_C \nabla \tilde{C}$ is explicitly demoted to a phenomenological, Newtonian-limit parametrisation and is not an independent gravitational mechanism.

With this normalisation in place, all subsequent use of the coherence field in galaxy rotation models, cosmology, or collapse dynamics can be expressed in terms of the dimensionless field $\tilde{C}$ and explicitly tracked dependence on (f, e, m), α_C and η_C. This ensures compatibility with the unified coherence dictionary, with a single, GR-based gravitational sector, and with an electromagnetic contribution that is included or neglected as a controlled approximation depending on the physical regime.