Curvature and Coherence in the Skyrme–Taiji Framework: A Unified Gravitational Formulation

Taiji–Skyrme Program

Abstract

This document develops a curvature-based gravitational formulation for the Skyrme–Taiji coherence field. The dynamical content is encoded in an $SU(2)$-valued field $U(x)$ whose nonlinear structure and topological charge support soliton-like excitations. The coherence scalar $C(x)$ serves as an order parameter quantifying local field alignment.

Gravity is treated exclusively as spacetime curvature sourced by the symmetric stress–energy tensor $T_{\mu\nu}[U]$. Coherence influences gravitational behaviour indirectly through its modulation of the energy density and pressures of the Skyrme–Taiji field. No coherence-based force law is assumed. The Newtonian limit, rotation-curve implications, emergent coherence-gradient dynamics, and fragmentation structures are developed within this consistent geometric framework.

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1 Introduction

The Skyrme–Taiji formulation models field configurations valued in $SU(2)$, capturing both topological and coherence-driven features of the Excitation–Potential Field system. Finite-energy boundary conditions compactify spatial infinity to S^3, enabling nontrivial soliton structures that serve as candidate matter-like excitations.

Coherence plays a central role in determining the spatial organization of these configurations. Its gradients correlate with regions of high or low energy concentration, and these correlations strongly influence effective dynamics. Gravitational behaviour, however, arises not from coherence directly but from the curvature generated by the field's stress–energy tensor. This document presents a complete and self-contained treatment of that curvature-based gravitational sector.

2 Field Content

2.1 The $SU(2)$ Field

The fundamental object is an $SU(2)$ field

$$U(x) \in SU(2), \qquad U^\dagger U = 1, \quad \det U = 1. \tag{1}$$

It is convenient to define right-invariant currents

$$R_\mu = U^\dagger \partial_\mu U, \tag{2}$$

which encode all dynamical information and satisfy $R_\mu^\dagger = -R_\mu$.

2.2 Coherence Scalar

The coherence scalar $C(x)$ measures local field ordering. Its explicit dependence on U and its derivatives may vary across models, but a schematic expansion is

$$C(x) = 1 - \alpha \,\mathrm{Tr}(R_i R_i) + \mathcal{O}(R^4), \qquad 0 \le C \le 1. \tag{3}$$

Regions with reduced coherence typically correspond to soliton cores or transition layers between ordered domains.

Throughout this document, the term "field" without further qualification refers to the $SU(2)$ configuration $U(x)$ and its associated coherence scalar $C(x)$. Electromagnetic degrees of freedom belong to the full Taiji–Skyrme framework through the $U(1)$ coherence connection A_μ and its field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The configurations analysed below are neutral and weakly magnetised: in the explicit solutions we therefore set $A_\mu = 0$ and $F_{\mu\nu} = 0$, so Maxwell fields do not enter the worked examples. Their gravitational effect is included whenever needed through the electromagnetic contribution to the total stress–energy tensor.

3 Lagrangian and Energetics

The base Lagrangian for static and dynamical configurations is

$$\mathcal{L} = \frac{f^2}{16}\mathrm{Tr}(R_\mu R^\mu) + \frac{1}{32e^2}\mathrm{Tr}([R_\mu, R_\nu][R^\mu, R^\nu]) - V(U), \tag{4}$$

where $V(U)$ ensures relaxation toward the vacuum configuration $U = \mathbf{1}$.

The energy density of a static field is

$$\mathcal{E} = \frac{f^2}{16}\mathrm{Tr}(R_i R_i) + \frac{1}{32e^2}\mathrm{Tr}([R_i, R_j][R_i, R_j]) + V(U). \tag{5}$$

Coarse-graining over a region of size L yields an effective coherence-modulated energy

$$\rho_{\mathrm{coh}}(x) = \frac{1}{V_L}\int_{|y-x|<L} \mathcal{E}(y)\, d^3y, \tag{6}$$

which defines an effective coherence–modulated energy density. In regimes where the Skyrme–Taiji sector dominates the stress–energy budget, this quantity provides the leading contribution to the effective gravitational mass density in the Newtonian limit, to be combined with baryonic, electromagnetic, and any additional components when constructing the full source.

4 Topological Structure

The baryon-like topological current is defined by

$$B^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2}\mathrm{Tr}\big(U^{-1}\partial_\nu U\; U^{-1}\partial_\rho U\; U^{-1}\partial_\sigma U\big)\,, \tag{7}$$

with total charge

$$B = \int d^3x\, B^0 \in \mathbb{Z}. \tag{8}$$

Configurations with $|B| = 1$ describe localized coherence knots; larger $|B|$ allow multi-knot or fragmented structures.

5 Stress–Energy Tensor

Variation of the action with respect to the metric produces the symmetric stress–energy tensor:

$$\begin{aligned} T_{\mu\nu} = {} & \frac{f^2}{8}\mathrm{Tr}(R_\mu R_\nu) - \eta_{\mu\nu}\frac{f^2}{16}\mathrm{Tr}(R_\alpha R^\alpha) \\ & + \frac{1}{8e^2}\mathrm{Tr}([R_\mu, R_\alpha][R_\nu, R^\alpha]) - \eta_{\mu\nu}\frac{1}{32e^2}\mathrm{Tr}\big([R_\alpha, R_\beta][R^\alpha, R^\beta]\big) - \eta_{\mu\nu}V(U). \end{aligned} \tag{9}$$

This tensor determines all gravitational behaviour through

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}. \tag{10}$$

Coherence influences gravity only through its effect on R_i and thereby on $T_{\mu\nu}$.

In the full Taiji–Skyrme framework, the Einstein tensor is sourced by the sum of all sector stress–energy tensors,

$$T^{\rm tot}_{\mu\nu} = T^{\rm (Sk)}_{\mu\nu} + T^{\rm (coh)}_{\mu\nu} + T^{\rm (EM)}_{\mu\nu} + T^{\rm (other)}_{\mu\nu}.$$

Here $T^{\rm (Sk)}_{\mu\nu}$ and $T^{\rm (coh)}_{\mu\nu}$ reduce, for the neutral sector considered in this document, to the expression in (9), $T^{\rm (EM)}_{\mu\nu}$ is the Maxwell stress–energy tensor associated with the $U(1)$ coherence connection A_μ, and $T^{\rm (other)}_{\mu\nu}$ collects any additional matter components. The electromagnetic contribution has the standard form

$$T^{\rm (EM)}_{\mu\nu} = \frac{1}{g^2_{\rm EM}} \left(F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4}\, g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right),$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In the neutral, uncharged configurations analysed below one sets $A_\mu = 0$ and $F_{\mu\nu} = 0$, so $T^{\rm (EM)}_{\mu\nu}$ does not contribute and $T^{\rm tot}_{\mu\nu}$ is represented by (9).

6 Spherical Configurations

For static, spherically symmetric configurations, the metric takes the form

$$ds^2 = A(r)\, dt^2 - B(r)\, dr^2 - r^2 d\Omega^2, \tag{11}$$

and the hedgehog ansatz for the field is

$$U(r) = \cos F(r)\, \mathbf{1} + i(\hat{x} \cdot \sigma) \sin F(r). \tag{12}$$

The stress–energy components reduce to an energy density $\rho(r)$, radial pressure $p_r(r)$, and tangential pressure $p_t(r)$. The Einstein equations are

$$\frac{1 - B^{-1}}{r^2} - \frac{B'}{rB^2} = \kappa \rho(r), \tag{13}$$

$$\frac{1 - B^{-1}}{r^2} + \frac{A'}{rAB} = \kappa p_r(r), \tag{14}$$

$$\text{(angular component)} = \kappa p_t(r). \tag{15}$$

7 Newtonian Limit

In weak fields and for slow motion,

$$A(r) = 1 + 2\Phi(r), \qquad B(r) = 1 - 2\Phi(r), \tag{16}$$

with $\Phi(r)$ the Newtonian gravitational potential. To leading order in Φ, the Einstein equations reduce to the Poisson equation

$$\nabla^2\Phi = 4\pi G\,\rho_{\rm eff}(x), \tag{17}$$

where the effective source is defined directly from the total stress–energy tensor,

$$\rho_{\rm eff}(x) \equiv \rho_{\rm tot}(x) = T^{\rm tot}_{00}(x) = \rho_{\rm baryon}(x) + \rho_{\rm coh}(x) + \rho_{\rm EM}(x) + \rho_{\rm other}(x). \tag{18}$$

Here

$$\rho_{\rm baryon} \equiv T^{\rm (baryon)}_{00}, \quad \rho_{\rm coh} \equiv T^{\rm (coh)}_{00}, \quad \rho_{\rm EM} \equiv T^{\rm (EM)}_{00}, \quad \rho_{\rm other} \equiv T^{\rm (other)}_{00}, \tag{19}$$

so that all sectors contributing to $T^{\rm tot}_{\mu\nu}$ appear on an equal footing in the Newtonian limit.

For the Skyrme–Taiji configurations treated in this document we focus on neutral, coherence–dominated solutions. In that regime baryonic, electromagnetic, and additional components can be neglected at the level of the examples, and the effective source reduces to the Skyrme–Taiji contribution

$$\rho_{\rm eff}(x) = T_{00}(x) = \frac{f^2}{16}\,{\rm Tr}(R_iR_i) + \frac{1}{32e^2}\,{\rm Tr}([R_i,R_j][R_i,R_j]) + V(U), \tag{20}$$

which coincides with the static energy density of the field introduced in the Lagrangian and energetics section. This potential Φ is the unique gravitational potential arising from the theory; no alternative potential identification is assumed.

Electromagnetic contribution to gravity

Electromagnetic fields enter the gravitational source through $\rho_{\rm EM}$ and through the spatial components of $T^{\rm (EM)}_{\mu\nu}$. The Maxwell sector associated with the coherence connection A_μ therefore gravitates in exactly the same way as any other field: its energy density, pressure, and anisotropic stresses contribute to the right-hand side of the Einstein equations.

8 Coherence and Emergent Dynamics

Although gravity does not originate from coherence gradients, coherence structure shapes the energy density, producing motion that may appear coherence-driven. For extended configurations, one often finds

$$\nabla C \propto \nabla{\rm Tr}(R_iR_i), \tag{21}$$

so that curvature wells align with coherence minima.

Effective radial motion of a soliton-like object obeys

$$m_{\rm eff}\frac{d^2r}{dt^2} = -\frac{\partial}{\partial r}\int d^3x\,\rho_{\rm eff}(x), \tag{22}$$

reflecting gravitational influence through curvature rather than direct coherence forces.

9 Rotation-Curve Behaviour

Extended coherence textures may yield energy-density profiles scaling like

$$\rho_{\text{eff}}(r) \sim \frac{1}{r^2}, \tag{23}$$

leading to the logarithmic potential

$$\Phi(r) \sim \ln r. \tag{24}$$

Circular velocity then satisfies

$$v^2(r) = r\Phi'(r) \approx \text{constant}, \tag{25}$$

naturally reproducing flat galactic rotation curves.

10 Fragmentation and Multi-Well Structure

Topological charge can fragment across space, especially on compact manifolds like T^3. Each fragment generates a localized curvature well arising from its stress–energy distribution. Interference between multiple wells produces complex gravitational landscapes relevant for the early stages of structure formation.

11 Numerical Implementation

Time evolution of the field can be studied using:

- finite-difference discretization of $U(x,t)$,
- adaptive solvers for the profile function $F(r,t)$,
- explicit computation of $T_{\mu\nu}$ on each timestep,
- weak-field or fully relativistic solvers for curvature.

These tools allow exploration of soliton stability, merger dynamics, and coherence-driven pattern formation.

11.1 Coherence–Preserving Jet Configurations

In the Skyrme–Taiji framework, gravity is purely geometric:

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}, \tag{26}$$

and coherence influences the motion of matter only through its contribution to the stress–energy tensor. There is no independent "coherence force law"; all effective forces arise from curvature sourced by $T_{\mu\nu}$. Nevertheless, certain stress–energy configurations can be singled out as *coherence–preserving* in a precise sense.

11.1.1 Coherence Balance and Flux

Let $C_{\mathrm{phys}}(x)$ denote the physical coherence density, as defined in the coherence dictionary and normalisation documents, and let Σ_t be a spatial slice with induced metric γ_{ij} and volume element $d^3x\sqrt{\gamma}$. The total coherence charge in a spatial region $\Omega_t \subset \Sigma_t$ is

$$Q_C[\Omega_t] \;=\; \int_{\Omega_t} C_{\mathrm{phys}}(t,\boldsymbol{x})\,\sqrt{\gamma}\,d^3x. \tag{27}$$

Conservation of total energy–momentum, $\nabla_\mu T^{\mu\nu}_{\mathrm{tot}} = 0$, together with the decomposition of $T^{\mu\nu}_{\mathrm{tot}}$ into a coherence sector and the remaining sectors, implies a local balance law for C_{phys} of the schematic form

$$\partial_t C_{\mathrm{phys}}(t,\boldsymbol{x}) + \nabla_i J^i_C(t,\boldsymbol{x}) = S_C(t,\boldsymbol{x}), \tag{28}$$

where

- J^i_C is a *coherence flux* density, defined in terms of the coherence–sector energy flux $T^{(\mathrm{coh})}_{0i}$ via a fixed calibration;
- S_C is a *source/sink* term encoding exchange of coherence between the coherence sector and the other sectors.

Integrating (28) over Ω_t yields

$$\frac{dQ_C[\Omega_t]}{dt} = \int_{\Omega_t} S_C\,\sqrt{\gamma}\,d^3x - \oint_{\partial\Omega_t} J^i_C\,d\Sigma_i, \tag{29}$$

where $d\Sigma_i$ is the directed surface element on the boundary $\partial\Omega_t$. Equation (29) is purely kinematical: it states that the rate of change of coherence in a region equals bulk production/destruction minus the net coherence flux through its boundary.

For a given spatial slice Σ_t with induced metric γ_{ij} and volume element $\sqrt{\gamma}\,d^3x$, we define the *total* coherence charge contained in a region $\Omega_t \subset \Sigma_t$ by

$$Q_C[\Omega_t] = \int_{\Omega_t} C_{\mathrm{phys}}(t,x)\,\sqrt{\gamma}\,d^3x, \tag{30}$$

where C_{phys} is the branch-summed physical coherence density introduced in (33). For the core, jet tubes, and environment we write

$$Q_C^{\mathrm{core}} = Q_C[\Omega_{\mathrm{core}}], \qquad Q_C^{\mathrm{jet}} = Q_C[\Omega^{(+)}_{\mathrm{jet}} \cup \Omega^{(-)}_{\mathrm{jet}}], \qquad Q_C^{\mathrm{env}} = Q_C[\Omega_{\mathrm{env}}], \tag{31}$$

so that $Q_C = Q_C^{\mathrm{core}} + Q_C^{\mathrm{jet}} + Q_C^{\mathrm{env}}$.

Similarly, the total coherence flux through a closed two-surface S_R is

$$F_C^{\mathrm{out}}(R) = \oint_{S_R} J^i_C(t,x)\,d\Sigma_i, \tag{32}$$

with J^i_C the branch-summed flux. Its decomposition into jet and equatorial components, $F_C^{\mathrm{jet}}(R)$ and $F_C^{\mathrm{eq}}(R)$, and the fluxes $F_C^{\mathrm{core}\to\mathrm{jet}}$ and $F_C^{\mathrm{core}\to\mathrm{env}}$ appearing in the balance equations are all defined with the same total J^i_C. The coherence-preserving inequalities that follow are therefore conditions on the *total* coherence content and total coherence flux of the system.

11.1.2 N-branch interpretation of coherence observables

In this document the symbols $C_{\rm phys}(x)$, $J^i_C(x)$ and $S_C(x)$ are understood as the *total* coherence density, flux and source, in the sense of the coherence dictionary and normalisation notes. The coherence sector admits a set of branches $\mathcal{B} = \{1, 2, \ldots, N\}$, labelled by b, with branch-resolved coherence densities $C^{(b)}_{\rm phys}(x)$ and associated fluxes $J^i_{C,b}(x)$ and source terms $S_{C,b}(x)$. The total observables entering the gravitational and balance equations are defined as the branch sums

$$C_{\rm phys}(x) = \sum_{b\in\mathcal{B}} C^{(b)}_{\rm phys}(x), \qquad J^i_C(x) = \sum_{b\in\mathcal{B}} J^i_{C,b}(x), \qquad S_C(x) = \sum_{b\in\mathcal{B}} S_{C,b}(x). \tag{33}$$

These relations are not new dynamical assumptions: they simply restate, in the N-branch language, that the coherence density and its flux are the macroscopic, branch-summed quantities defined in the coherence dictionary and normalisation documents. All gravitational effects of coherence still enter exclusively through the total stress–energy tensor $T^{\rm tot}_{\mu\nu}$.

11.1.3 Core, Jet and Environment Decomposition

Consider a stationary or quasi–stationary configuration that is approximately axisymmetric around an axis $\hat{\boldsymbol{z}}$. We decompose the spatial slice into three regions:

1. a *core* region $\Omega_{\rm core}$, containing the compact object and inner coherence structure;
2. two *jet tubes* $\Omega^{(\pm)}_{\rm jet}$, narrow columns extending along $\pm\hat{\boldsymbol{z}}$ away from the core;
3. an *environment* region $\Omega_{\rm env} = \Sigma_t \setminus (\Omega_{\rm core} \cup \Omega^{(+)}_{\rm jet} \cup \Omega^{(-)}_{\rm jet})$.

We define the partial coherence charges

$$Q^{\rm core}_C = Q_C[\Omega_{\rm core}], \qquad Q^{\rm jet}_C = Q_C[\Omega^{(+)}_{\rm jet} \cup \Omega^{(-)}_{\rm jet}], \qquad Q^{\rm env}_C = Q_C[\Omega_{\rm env}], \tag{34}$$

so that $Q_C = Q^{\rm core}_C + Q^{\rm jet}_C + Q^{\rm env}_C$. Applying (29) to $\Omega_{\rm core}$ gives

$$\frac{dQ^{\rm core}_C}{dt} = \int_{\Omega_{\rm core}} S_C \,\sqrt{\gamma}\, d^3x - \oint_{\partial\Omega_{\rm core}} J^i_C \, d\Sigma_i, \tag{35}$$

where the boundary flux can be split into contributions towards the jet tubes and towards the environment,

$$\oint_{\partial\Omega_{\rm core}} J^i_C \, d\Sigma_i = F^{\rm core\to jet}_C + F^{\rm core\to env}_C. \tag{36}$$

11.1.4 Definition of Coherence–Preserving Jets

We now give a purely geometric and flux–based definition of a *coherence–preserving jet configuration*, without introducing any new forces beyond curvature sourced by $T_{\mu\nu}$.

Jet–like flux. Fix a large surface S_R enclosing the core and define the total outward coherence flux

$$F_C^{\rm out}(R) = \oint_{S_R} J_C^i \, d\Sigma_i. \tag{37}$$

Let $F_C^{\rm jet}(R)$ be the flux through two polar caps of opening angle $\theta_{\rm jet}$ around $\pm\hat{\boldsymbol{z}}$, and let $F_C^{\rm eq}(R) = F_C^{\rm out}(R) - F_C^{\rm jet}(R)$ be the flux through the complementary equatorial belt. A configuration is said to be *jet–like* in the coherence sense if, for sufficiently large R,

$$\frac{\left|F_C^{\rm jet}(R)\right|}{|F_C^{\rm out}(R)|} \geq \eta_{\rm col}, \qquad \eta_{\rm col} \lesssim 1, \tag{38}$$

i.e. most outward coherence flux is confined to narrow cones along the axis. In addition, $C_{\rm phys}$ should be core–dominated compared to the environment,

$$\sup_{\Omega_{\rm env}\setminus\Omega_{\rm jet}} C_{\rm phys} \ll \sup_{\Omega_{\rm core}} C_{\rm phys}. \tag{39}$$

Coherence–preserving core. Let $\tau_{\rm dyn}$ be a characteristic dynamical timescale of the core (e.g. a rotation or free–fall time). We say that the core is *coherence–preserving* if, on timescales $\Delta t \sim \tau_{\rm dyn}$, the following conditions hold:

$$\text{(i) quasi–stationary coherence:} \quad \left|\frac{1}{Q_C^{\rm core}}\frac{dQ_C^{\rm core}}{dt}\right| \ll \frac{1}{\tau_{\rm dyn}}, \tag{40}$$

$$\text{(ii) flux–dominated balance:} \quad \left|\int_{\Omega_{\rm core}} S_C \sqrt{\gamma}\, d^3x\right| \ll \left|F_C^{\rm core\to jet}\right| + |F_C^{\rm core\to env}|, \tag{41}$$

$$\text{(iii) jet–dominated export:} \quad \left|F_C^{\rm core\to jet}\right| \gg |F_C^{\rm core\to env}|. \tag{42}$$

Condition (40) states that the relative change of core coherence per dynamical time is small; (41) states that core coherence is transported rather than created or destroyed in bulk; and (42) states that this transport proceeds primarily along the jet tubes, not isotropically.

Coherence–preserving jet class. A configuration is said to belong to the *coherence–preserving jet class* if it is jet–like in the sense of (38) and satisfies (40)–(42) for some reasonable choice of $\Omega_{\rm core}$ and $\tau_{\rm dyn}$. By construction, such configurations:

- maintain an approximately constant coherence content in the core;
- offload excess energy and stress primarily through narrow, curvature–sourcing outflows;
- do so without invoking any additional force law beyond general relativity.

In astrophysical terms, black hole–disk systems with relativistic jets are natural candidates for this class: the jets act as structured channels through which coherence–bearing stress–energy is exported, while the inner coherence structure remains metastable and continues to source curvature consistently with the Einstein equations.

Branch-resolved jet diagnostics

In the N-branch framework, it is often useful to track how individual branches contribute to the core and jet coherence budgets. For any branch $b \in \mathcal{B}$ we may define a branch-resolved coherence charge in the core region by

$$Q^{\mathrm{core}}_{C,b} = \int_{\Omega_{\mathrm{core}}} C^{(b)}_{\mathrm{phys}}(t,x)\,\sqrt{\gamma}\,d^3x, \tag{43}$$

and similarly a branch-resolved jet flux

$$F^{\mathrm{core}\to\mathrm{jet}}_{C,b} = \int_{\partial\Omega_{\mathrm{core}}\cap\Omega_{\mathrm{jet}}} J^i_{C,b}(t,x)\,d\Sigma_i, \tag{44}$$

with analogous expressions for $Q^{\mathrm{jet}}_{C,b}$, $Q^{\mathrm{env}}_{C,b}$ and $F^{\mathrm{core}\to\mathrm{env}}_{C,b}$. One may then formulate branch-resolved coherence-preserving conditions by applying the same inequalities as in the total case to any chosen subset of branches.

In the present document the classification of coherence-preserving jet configurations is stated in terms of the *total* coherence charge and fluxes, as this is the quantity directly tied to the gravitational source $T^{(\mathrm{coh})}_{\mu\nu}$. Branch-resolved diagnostics such as (43)–(44) are intended as an

11.2 Phenomenological outlook and observational constraints

The coherence-preserving jet class defined above provides a structural way to encode black-hole + disc + jet systems in the Taiji–Skyrme framework. From the gravitational point of view, all such configurations are described by solutions of the Einstein equations

$$G_{\mu\nu} = 8\pi G\,T^{\mathrm{tot}}_{\mu\nu}, \tag{45}$$

with $T^{\mathrm{tot}}_{\mu\nu}$ built from the baryonic, coherence, Maxwell and any additional sectors according to the normalisation document. The N-branch structure controls how the coherence contribution $T^{(\mathrm{coh})}_{\mu\nu}$ is distributed between the core, jet tubes and environment, but does not introduce any new gravitational degrees of freedom.

In the vicinity of astrophysical black holes, any Taiji-induced deformation of the metric must remain parametrically small in order to be compatible with current horizon-scale imaging constraints (ngEHT/BHEX) on the size and shape of the black-hole shadow. In practice this means that, when one writes the spacetime metric in a parameterised-deformation scheme (such as the KRZ family discussed in strategic reference [SR-GRV-001]), the effective deformation parameters induced by the coherence sector and its branch structure must lie within the observational bounds extracted from shadow measurements and related observables.

Branch-dependent core/jet configurations in the present framework map onto such parameterised deviations through their effect on the near-horizon stress–energy tensor. Different choices of branch weights and branch activity in the core and jet regions correspond to different patterns of anisotropic stresses and energy flows, which in turn induce small deformations of the background Kerr geometry. The coherence-preserving jet conditions therefore

provide a way to organise these configurations while ensuring that any resulting metric deviations remain in the regime allowed by current and future horizon-scale observations. optional refinement for future, more detailed analyses.

12 Conclusion

This document establishes a self-contained, curvature-based treatment of gravitational phenomena in the Skyrme–Taiji coherence framework. Coherence influences gravity only through its modulation of the stress–energy tensor, never as a force or potential. Rotation curves, fragmentation, and emergent coherence-gradient dynamics arise naturally from curvature sourcing.

The resulting theory forms a consistent foundation for astrophysical applications and future extension into cosmological modelling.