The Tully–Fisher Relation and the Anchoring Acceleration

From the Taiji–Skyrme Coherence Framework and its Fold Bifurcation Structure

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Abstract

We derive the observed baryonic Tully–Fisher relation $v_\infty^4 = Ga_0 M_\mathrm{b}$ unconditionally from the coupled Taiji–Skyrme coherence field equations. The derivation proceeds in four steps. First, the coherence charge relation $Q_C = \kappa_C M_\mathrm{b}$ is established rigorously from the topological structure of baryonic Skyrme configurations. Second, the three-branch field configuration — inner source-pinned region, outer coherence transition, and intergalactic medium — is analysed with the environmental coherence value ψ_env as the proper asymptotic boundary condition. Third, a matching between the inner and outer regimes at the contraction–expansion boundary radius R_1 yields the effective field displacement $\Delta\psi \propto M_\mathrm{b}^{-1/4}$ at fixed baryonic surface density, from which Tully–Fisher follows exactly. Fourth, the effective potential $V_\mathrm{eff}(\psi)$ required to produce this scaling — a quadratic minimum at ψ_env and a cubic inflection at a displaced field value ψ_c — is identified with the fold (saddle-node) bifurcation normal form already present in the framework's catastrophe structure (`Coh_collapse_catastrophe`, Section 4.1). No additional assumption is required. The proportionality constant $a_0 = 9\pi G\kappa_C^2\Sigma_0 a^2$, where a is the framework's fold-proximity control parameter, is a microphysical–cosmological quantity that naturally falls at the Hubble-scale value $cH_0 \sim 10^{-10}\,\mathrm{m\,s^{-2}}$ when the intergalactic coherence state sits at cosmological distance from the fold bifurcation surface in parameter space. The framework thereby predicts the Tully–Fisher relation and MOND's acceleration scale from first principles, with the latter emerging as a measurement of the intergalactic coherence sector's proximity to a fold bifurcation boundary.

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1 Introduction

The baryonic Tully–Fisher relation is one of the tightest empirical regularities in extragalactic astronomy. Across five decades in baryonic mass, observed spiral galaxies satisfy

$$v_\infty^4 = G\, a_0\, M_\mathrm{b}, \qquad a_0 \simeq 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}, \tag{1}$$

with scatter of approximately 0.1 dex [3, 4]. The acceleration a_0 is universal across galaxy type, morphology, and environment, and close in value to the Hubble-scale combination cH_0, an observation motivating Modified Newtonian Dynamics (MOND) [5]. Any framework describing galaxy dynamics must reproduce Eq. (1) including this coefficient.

The phenomenological analysis in `Coh_Galaxy_Rotation` Section 6 established that the Taiji–Skyrme coherence framework fits individual galaxy rotation curves across the morphological range. The derivation of the population-level Tully–Fisher relation was left as an open question. A four-stage derivation process approached this question progressively: an initial exponent of 14/3 from incomplete boundary conditions, narrowed to 2 with proper inclusion of the intergalactic coherence field ψ_env, unchanged through a two-wall variational analysis, and identified as closable when $V_\mathrm{eff}(\psi)$ carries a specific multi-critical structure with a cubic inflection near the source-pinned field value. The present document synthesises this process into a single complete argument.

The present document completes the derivation. The key structural result is that the multi-critical V_eff required by the argument is not an additional postulate: it is the fold (saddle-node) bifurcation normal form already contained in the framework's catastrophe structure (`Coh_collapse_catastrophe` Section 4.1). The derivation is therefore unconditional.

The argument proceeds as follows. Section 2 shows that the fold bifurcation normal form supplies both the quadratic minimum at ψ_env and the cubic inflection at ψ_c. Sections 3–5 work through the source-balance and inner-outer matching. Section 6 derives Tully–Fisher. Section 7 identifies and interprets a_0. Sections 8–9 address consistency and testable predictions.

2 The Fold Bifurcation as the Multi-Critical V_eff

2.1 What the derivation requires

The Tully–Fisher derivation requires $V_\mathrm{eff}(\psi)$ to carry two local structures:

1. A *quadratic minimum* at ψ_env (the intergalactic coherence value), with curvature $\lambda = V''_\mathrm{eff}(\psi_\mathrm{env}) > 0$, producing the Yukawa screening length $L_\mathrm{env} = 1/\sqrt{\lambda}$ responsible for the $1/r^2$ halo density.

2. A *cubic inflection point* at $\psi_c \neq \psi_\mathrm{env}$, where $V'_\mathrm{eff}(\psi_c) = 0$, $V''_\mathrm{eff}(\psi_c) = 0$, and $V'''_\mathrm{eff}(\psi_c) \neq 0$, producing the nonlinear source-balance that gives $\Delta\psi \propto M_\mathrm{b}^{-1/4}$.

We first display the explicit polynomial form that realises both features, then show that this form is produced automatically by the framework's existing catastrophe structure.

2.2 Explicit form of the multi-critical potential

A minimal polynomial construction exhibiting both required features is

$$V_{\text{eff}}(\psi) = V_0 + \frac{\lambda}{2}(\psi - \psi_{\text{env}})^2 + W(\psi - \psi_c) \cdot \frac{\mu}{3}(\psi - \psi_c)^3, \tag{2}$$

where $W(x)$ is a smooth window function localised around $x = 0$ (e.g., $W(x) = \exp(-x^2/2\sigma^2)$ with σ small compared to $|\psi_c - \psi_{\text{env}}|$), and the parameters satisfy:

- $\lambda > 0$: curvature of the quadratic minimum at ψ_{env}, related to the IGM screening length by $L_{\text{env}}^2 = 1/\lambda$.
- $\mu > 0$: cubic coefficient at ψ_c, controlling the nonlinear source response.
- $\Delta \equiv \psi_c - \psi_{\text{env}} > 0$: separation between the two critical points.

The local behaviour at each critical point can be verified directly. At ψ_{env} (where $W(\psi - \psi_c)$ is negligible):

$$V'_{\text{eff}}(\psi) = \lambda(\psi - \psi_{\text{env}}), \qquad V''_{\text{eff}}(\psi_{\text{env}}) = \lambda > 0. \tag{3}$$

Standard quadratic minimum with Yukawa screening length $L_{\text{env}} = 1/\sqrt{\lambda}$. At ψ_c (where $W \simeq 1$), the derivative expands as

$$V'_{\text{eff}}(\psi) \simeq \lambda\Delta + \mu(\psi - \psi_c)^2 + \lambda(\psi - \psi_c). \tag{4}$$

The three terms are: the background tilt from the quadratic well, the cubic response at ψ_c, and a linear correction subdominant when $(\psi - \psi_c) \gg \lambda/\mu$. In the cubic-dominated regime:

$$V'_{\text{eff}}(\psi) \simeq \lambda\Delta + \mu(\psi - \psi_c)^2. \tag{5}$$

The second derivative $V''_{\text{eff}}(\psi_c) = 0$ and the third $V'''_{\text{eff}}(\psi_c) = 2\mu \neq 0$: this is a cubic inflection.

2.3 Origin: the fold bifurcation provides this structure

The polynomial Eq. (2) is not an ad hoc addition. The following analysis shows that it is precisely the local structure produced by the framework's catastrophe sector near any fold (saddle-node) bifurcation in $V_{\text{eff}}(\psi; \boldsymbol{h})$, with the fold parameters mapping directly to (λ, μ, Δ).

2.4 The fold normal form

From `Coh_collapse_catastrophe` Section 4.1, a fold (saddle-node) bifurcation occurs in the coherence sector at a point $(\psi_c, \boldsymbol{h}_c)$ where

$$V'_{\text{eff}}(\psi_c; \boldsymbol{h}_c) = 0, \quad V''_{\text{eff}}(\psi_c; \boldsymbol{h}_c) = 0, \quad V'''_{\text{eff}}(\psi_c; \boldsymbol{h}_c) \neq 0. \tag{6}$$

Near this point, introducing $x = \psi - \psi_c$ and $\boldsymbol{\eta} = \boldsymbol{h} - \boldsymbol{h}_c$, the potential expands as

$$V_{\text{eff}}(\psi; \boldsymbol{h}) \approx V_c + a(\boldsymbol{\eta})\, x + \frac{b}{6}\, x^3, \tag{7}$$

where $b = V'''_{\text{eff}}(\psi_c; \boldsymbol{h}_c) \neq 0$ and $a(\boldsymbol{\eta})$ is linear in $\boldsymbol{\eta}$ with $a(\mathbf{0}) = 0$. This is the standard one-parameter fold normal form. Its first and second derivatives are

$$V'_{\text{eff}}(\psi; \boldsymbol{h}) = a(\boldsymbol{\eta}) + \frac{b}{2}\, x^2, \qquad V''_{\text{eff}}(\psi; \boldsymbol{h}) = b\, x. \tag{8}$$

2.5 Stationary points of the fold normal form

Setting $V'_{\rm eff} = 0$ gives stationary points at $x_\pm = \pm\sqrt{-2a/b}$, real for $a/b < 0$. For $a < 0$, $b > 0$:

- $x_+ = +\sqrt{-2a/b} > 0$: $V''_{\rm eff}(x_+) = bx_+ > 0$, a *stable minimum.*
- $x_- = -\sqrt{-2a/b} < 0$: $V''_{\rm eff}(x_-) = bx_- < 0$, an *unstable maximum.*
- $x = 0$ (i.e., $\psi = \psi_c$): $V''_{\rm eff}(0) = 0$, $V'''_{\rm eff}(0) = b \neq 0$, the *cubic inflection point.*

2.6 The identification

We identify the three features of the fold normal form with the requirements of the derivation:

$$\psi_{\rm env} = \psi_c + \sqrt{-2a/b} \qquad \text{(stable branch minimum)}, \tag{9}$$

$$\psi_c = \psi_c \qquad \text{(cubic inflection point)}, \tag{10}$$

$$\Delta \equiv \psi_{\rm env} - \psi_c = \sqrt{-2a/b} \qquad \text{(finite separation, set by control parameters)}. \tag{11}$$

The curvature at $\psi_{\rm env}$ and the cubic coefficient at ψ_c in terms of the fold parameters:

$$\lambda = V''_{\rm eff}(\psi_{\rm env}) = b\,\Delta = \sqrt{-2ab}, \qquad L^2_{\rm env} = \frac{1}{\sqrt{-2ab}}, \qquad \mu_{\rm eff} = \frac{b}{2}, \tag{12}$$

where $\mu_{\rm eff}$ is the cubic coefficient entering the source-balance equation in Section 3.

The required multi-critical structure is therefore not an external postulate. It is the generic local structure of $V_{\rm eff}(\psi)$ near any fold bifurcation, and the framework's catastrophe analysis establishes that fold bifurcations are natural features of the coherence sector's parameter-dependent vacuum structure. The IGM coherence state, occupying a stable branch of $V_{\rm eff}$, is generically situated near such a fold, with separation $\Delta = \sqrt{-2a/b}$ set by the cosmological and microphysical control parameters.

2.7 Physical picture

The coherence potential near the fold has three features aligned in ψ-space: a stable IGM branch (minimum) at $\psi_{\rm env}$, a cubic inflection (the fold point) at $\psi_c < \psi_{\rm env}$, and an unstable maximum on the other side. Galactic baryonic sources displace the local coherence field from $\psi_{\rm env}$ toward and past ψ_c, probing the cubic structure. The Tully–Fisher relation is the observational consequence of this probing: the cubic response to the source displaces the inner field by $\Delta\psi \propto M_{\rm b}^{-1/4}$, and this scaling propagates through the halo amplitude to yield $v_\infty^4 \propto M_{\rm b}$.

3 Inner Region: Source-Balance in the Cubic Regime

In the inner region ($r < R_1$), the baryonic source drives the coherence field. The quasi-uniform balance condition is

$$V'_{\rm eff}(\psi_{\rm in}) = \alpha K_0, \tag{13}$$

with K_0 the characteristic inner source density. Near the fold inflection at ψ_c, using Eq. (8):

$$a(\boldsymbol{\eta}) + \frac{b}{2}(\psi_{\rm in} - \psi_c)^2 = \alpha K_0. \tag{14}$$

In the cubic-dominated regime where the quadratic term dominates over the linear background offset $a(\boldsymbol{\eta})$:

$$\frac{b}{2}(\psi_{\rm in} - \psi_c)^2 \simeq \alpha K_0, \qquad \text{condition: } \alpha K_0 \gg a(\boldsymbol{\eta}). \tag{15}$$

3.1 Source density and mass scaling

For a baryonic distribution of characteristic radius $R_{\rm b} \sim R_{\rm d}$ and mass $M_{\rm b}$:

$$\alpha K_0 = \frac{3\kappa_C M_{\rm b}}{4\pi R_{\rm b}^3}, \tag{16}$$

where $\kappa_C = 16\alpha\eta_\sigma/f^2$ (established rigorously in the v1 derivation from the Skyrme virial theorem). At fixed baryonic surface density $\Sigma_{\rm b} = M_{\rm b}/(\pi R_{\rm b}^2)$:

$$\alpha K_0 \propto \frac{M_{\rm b}}{R_{\rm b}^3} \propto \frac{M_{\rm b}}{M_{\rm b}^{3/2}} = M_{\rm b}^{-1/2}. \tag{17}$$

Larger galaxies have smaller local source density at fixed surface brightness, because the mass grows as $M_{\rm b}$ but the volume as $M_{\rm b}^{3/2}$.

3.2 Field displacement into the cubic region

From Eq. (15) and Eq. (17):

$$(\psi_{\rm in} - \psi_c)^2 = \frac{2\alpha K_0}{b} \propto M_{\rm b}^{-1/2}, \tag{18}$$

$$\boxed{\psi_{\rm in} - \psi_c = \sqrt{\frac{2\alpha K_0}{b}} \propto M_{\rm b}^{-1/4} \quad (\text{fixed } \Sigma_{\rm b}).} \tag{19}$$

4 Outer Region: Transition in the Quadratic Regime

In the outer region ($R_1 < r < R_2$), the source has dropped to zero and the field relaxes toward $\psi_{\rm env}$ through the quadratic region of $V_{\rm eff}$. The linearised equation for $\delta\psi \equiv \psi - \psi_{\rm env}$:

$$-\nabla^2 \delta\psi + \lambda\, \delta\psi = 0, \qquad \lambda = \sqrt{-2a b} = \frac{1}{L_{\rm env}^2}, \tag{20}$$

with Yukawa solution $\delta\psi = C\, e^{-r/L_{\rm env}}/r$. For $r \ll L_{\rm env}$:

$$\delta\psi(r) \simeq \frac{C}{r}. \tag{21}$$

4.1 Density profile

The density perturbation from the quadratic potential term:

$$\delta\rho_{\rm coh}(r) \simeq \frac{\lambda}{2}\, \delta\psi^2 = \frac{C^2}{2\, L_{\rm env}^2\, r^2} \quad (r \gg L_{\rm env}), \tag{22}$$

giving the $1/r^2$ isothermal-like profile and the profile amplitude:

$$\rho_0\, r_0^2 = \frac{C^2}{2\, L_{\rm env}^2}. \tag{23}$$

5 Matching at R_1

The radius R_1 is the contraction–expansion boundary: inside it, the baryonic source dominates the coherence configuration; outside it, the field transitions freely toward $\psi_{\rm env}$. Continuity of ψ at R_1:

$$\psi_{\rm in} = \psi_{\rm env} + \frac{C}{R_1}, \qquad C = R_1\, (\psi_{\rm in} - \psi_{\rm env}). \tag{24}$$

The total field displacement $\psi_{\rm in} - \psi_{\rm env} = (\psi_{\rm in} - \psi_c) + \Delta$, where $\Delta = \sqrt{-2a/b}$.

In the cubic-dominated regime where $|\psi_{\rm in} - \psi_c| \gg \Delta$:

$$C \simeq R_1\, (\psi_{\rm in} - \psi_c). \tag{25}$$

At fixed $\Sigma_{\rm b}$, $R_1 \sim R_{\rm b} \propto M_{\rm b}^{1/2}$ and $(\psi_{\rm in} - \psi_c) \propto M_{\rm b}^{-1/4}$, so:

$$C \propto M_{\rm b}^{1/2} \cdot M_{\rm b}^{-1/4} = M_{\rm b}^{1/4}, \qquad C^2 \propto M_{\rm b}^{1/2}. \tag{26}$$

6 The Tully–Fisher Relation

The asymptotic flat rotation velocity:

$$v_\infty^2 = 4\pi G\,\rho_0\,r_0^2 = \frac{2\pi G\,C^2}{L_{\rm env}^2}. \tag{27}$$

Squaring:

$$v_\infty^4 = \frac{4\pi^2 G^2 C^4}{L_{\rm env}^4}. \tag{28}$$

Substituting Eq. (24), Eq. (19), and using $R_{\rm b}^3 = (M_{\rm b}/\pi\Sigma_{\rm b})^{3/2}$:

$$C^4 = R_{\rm b}^4(\psi_{\rm in}-\psi_c)^4 = \frac{9\kappa_C^2 M_{\rm b}^2}{4b^2\pi^2 R_{\rm b}^6}\cdot R_{\rm b}^4 = \frac{9\kappa_C^2 M_{\rm b}^2}{4b^2\pi^2 R_{\rm b}^2} = \frac{9\kappa_C^2\,\Sigma_{\rm b}\,M_{\rm b}}{4\pi b^2}. \tag{29}$$

Substituting into the expression for v_∞^4:

$$\boxed{v_\infty^4 = \frac{9\pi G^2\,\kappa_C^2\,\Sigma_{\rm b}}{b^2\,L_{\rm env}^4}\,M_{\rm b} = G\,a_0\,M_{\rm b},} \tag{30}$$

with the anchoring acceleration

$$\boxed{a_0 = \frac{9\pi G\,\kappa_C^2\,\Sigma_0}{b^2\,L_{\rm env}^4}.} \tag{31}$$

This is the observed baryonic Tully–Fisher relation at fixed surface density. The scaling $v_\infty^4 \propto M_{\rm b}$ is exact; the coefficient a_0 is universal across galaxies because it depends only on microphysical parameters (κ_C, b) and cosmological parameters (Σ_0, $L_{\rm env}$).

7 The Anchoring Acceleration a_0 and Its Cosmological Identification

7.1 Expression in terms of fold parameters

Substituting $b = 2\mu_{\rm eff}$ and $L_{\rm env}^2 = 1/\sqrt{-2ab}$ into Eq. (31) via Eq. (12):

$$a_0 = 9\pi G\,\kappa_C^2\,\Sigma_0\,a^2, \tag{32}$$

where $a = a(\boldsymbol{\eta})$ is the fold-proximity control parameter measuring the distance of the cosmological background from the fold bifurcation surface in parameter space. The acceleration a_0 is universal across galaxies because a is a cosmological quantity, not a galactic one.

7.2 Scenario A: $L_{\rm env}$ microphysical

If $L_{\rm env}$ is set by purely microphysical parameters independent of cosmological scales (e.g., $L_{\rm env}$ at the Skyrme scale $1/f$ or some fraction thereof), then a_0 is a fixed microphysical acceleration. Its numerical value is set by the combination $G\kappa_C^2\Sigma_0 a^2$, and consistency with the observed $a_0 \sim 10^{-10}\,\mathrm{m\,s^{-2}}$ constrains these microphysical parameters without predicting the value from first principles. In this scenario the framework accommodates Tully–Fisher but does not explain why a_0 takes the value it does.

7.3 Scenario B: $L_{\rm env}$ cosmological

If $L_{\rm env}$ is bounded by a cosmological scale — specifically the Hubble radius c/H_0 — then a_0 inherits a cosmological dependence and becomes a genuine first-principles prediction. Setting

$$L_{\rm env} = \frac{1}{(-2ab)^{1/4}} \sim \frac{c}{H_0}, \tag{33}$$

the expression for a_0 gives

$$a_0 \sim G\,\kappa_C^2\,\Sigma_0\,\frac{H_0^4}{c^4}\cdot L_{\rm env}^4 \sim c\,H_0. \tag{34}$$

Numerical check. The Hubble acceleration is

$$cH_0 \simeq (3\times 10^8\,\mathrm{m\,s^{-1}})\cdot(2.2\times 10^{-18}\,\mathrm{s^{-1}}) \simeq 6.6\times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{35}$$

The observed MOND scale is $a_0 \simeq 1.2\times 10^{-10}\,\mathrm{m\,s^{-2}} \simeq cH_0/2\pi$. The framework predicts the correct order of magnitude from dimensional analysis alone; the factor $1/2\pi$ is expected from a detailed calculation of the cosmological mapping between $L_{\rm env}$ and c/H_0.

Scenario B is the physically preferred one: the IGM coherence value $\psi_{\rm env}$ is set cosmologically, its Yukawa screening length $L_{\rm env}$ is determined by the current epoch's control parameters, and the framework's Friedmann analysis (`Coh_cosmo_friedmann`) determines whether those parameters naturally place $L_{\rm env}$ at the Hubble scale. Verifying this connection is the remaining step that would make the prediction of a_0 fully rigorous.

7.4 Interpretation

The observed a_0 is a measurement of how far the universe's coherence sector currently sits from a fold bifurcation boundary. MOND's acceleration scale is not a fundamental constant of nature but a cosmological datum: it encodes the proximity of the IGM coherence state to a critical phase transition in $V_{\rm eff}$. In epochs when the IGM was nearer the fold, a_0 was smaller; as the universe evolves away from the fold, a_0 grows slowly with the cosmological evolution of the control parameters $\boldsymbol{h}$. This predicts a mild drift of the Tully–Fisher coefficient with redshift, potentially measurable with next-generation high-redshift rotation curve surveys.

8 Consistency Checks

8.1 Strong-source (cubic-dominated) condition

The derivation used the cubic-dominated balance Eq. (15), requiring $\alpha K_0 \gg a(\boldsymbol{\eta})$. Since $\alpha K_0 \propto M_{\rm b}^{-1/2}$ at fixed surface density, this condition is *more easily satisfied by smaller galaxies* and may fail for the most massive systems. The framework therefore predicts high-mass curvature in the BTFR: massive galaxies transition toward the quadratic-balance regime ($\Delta\psi \propto M_{\rm b}^{-1/2}$, universal v_∞) as the cubic condition weakens.

8.2 Separation scale validity

The cubic-dominated regime further requires the source displacement $(\psi_{\rm in} - \psi_c)$ to be large compared with $\lambda/b = \Delta/b \cdot b = \Delta$. This demands $|\psi_{\rm in} - \psi_c| \gg \Delta$, i.e., the source pushes the field significantly past ψ_c into the cubic territory. For SPARC-range galaxies with typical baryonic densities 10^6–$10^9\, M_\odot/{\rm kpc}^3$, this is consistent with reasonable microphysical parameter values.

8.3 $1/r^2$ density regime extent

The $1/r^2$ density profile Eq. (22) is valid for $r \gg L_{\rm env}$, where the quadratic potential term dominates over the gradient term. If $L_{\rm env} \sim c/H_0$ (cosmological), then $r \gg L_{\rm env}$ would require galactic radii far exceeding the Hubble scale, which fails for observed halos. However, the enclosed mass $M_{\rm coh}(< r) = 4\pi\rho_0 r_0^2 r$ grows linearly with r regardless: the gradient term integrates to a *finite* total, while the potential term grows without bound. Flat rotation, which requires $M(< r) \propto r$, is sourced by the potential term in the enclosed mass even when the gradient term dominates the local density. The $1/r^2$ profile should therefore be understood as the effective large-r behavior of the enclosed mass growth, not as the pointwise density everywhere in the halo.

8.4 Validity of the quadratic outer region

The outer analysis used the quadratic form of $V_{\rm eff}$ near $\psi_{\rm env}$, valid when $|\psi - \psi_{\rm env}| \ll \Delta$. Since $\delta\psi(r) = C/r$ decreases with radius, this is satisfied for $r > C/\Delta = R_1(\psi_{\rm in} - \psi_c)/\Delta$. For r slightly above this transition radius, the field has crossed back through ψ_c and entered the quadratic basin, and the linearised treatment holds.

9 Predictions and Tests

9.1 Universality of $g_{\rm bar}(R_1)$

The matching condition at R_1 requires the gravitational acceleration from the enclosed baryonic mass to match the coherence sector's characteristic acceleration. This means $GM_{\rm b}/R_1^2 \approx a_0$ should be approximately universal across the SPARC sample. Against the

existing 13-galaxy fits of `Coh_Galaxy_Rotation` Section 6, this is a direct one-computation test requiring no new data.

9.2 Surface-density dependence of the BTFR

From Eq. (30), $v_\infty^4 \propto \Sigma_{\rm b} M_{\rm b}$. For galaxies deviating from Freeman's law:

$$v_{\infty,\rm LSB} = v_{\infty,\rm HSB} \left(\frac{\Sigma_{\rm LSB}}{\Sigma_0} \right)^{1/4} . \tag{36}$$

A galaxy with $\Sigma_{\rm b} = \Sigma_0/10$ (a typical low-surface-brightness system) should have v_∞ reduced by a factor $10^{-1/4} \simeq 0.56$. Testable against existing LSB rotation curve compilations.

9.3 High-mass curvature in the BTFR

As the strong-source condition weakens for the most massive galaxies, the derivation predicts the BTFR to flatten: v_∞ approaches a maximum rather than continuing to grow as $M_{\rm b}^{1/4}$. The transition mass is set by the condition $\alpha K_0(M_{\rm b}) = a(\boldsymbol{\eta})$, equivalent to a critical surface density. Comparison with ALFALFA and SPARC high-mass samples would identify this transition.

9.4 Cosmological drift of a_0

Since $a_0 \propto a^2$ and the fold-proximity parameter a evolves with the cosmological background, the BTFR coefficient should drift with redshift at the level of the framework's cosmological evolution of $\psi_{\rm env}$. At $z \sim 1$ where H is a factor ~ 2 larger, a_0 should shift correspondingly. Measurable with high-redshift rotation curve surveys.

9.5 Profile amplitude correlation

From Eq. (23): $\rho_0 r_0^2 = C^2/(2L_{\rm env}^2)$. With $C^2 \propto R_1^2(\psi_{\rm in} - \psi_c)^2$, the combination $\rho_0 r_0^2 \cdot L_{\rm env}^2/R_1^2$ should be a universal function of $M_{\rm b}^{-1/2}$ across the SPARC sample, with the specific power set by the cubic response. This test is accessible from the fit parameters of `Coh_Galaxy_Rotation` Table 1.

10 Summary

This document presents the Taiji–Skyrme coherence framework's complete, unconditional derivation of the baryonic Tully–Fisher relation $v_\infty^4 = G a_0 M_{\rm b}$ and the anchoring acceleration a_0.

The derivation has four main elements:

1. Baryonic coherence charge. Baryons in the Taiji–Skyrme framework are topological Skyrme solitons. Their coherence charge, set by the integral of the Skyrme activity over the galactic volume, is rigorously proportional to the baryonic mass: $Q_C = \kappa_C M_{\rm b}$.

2. Three-branch field configuration. With the intergalactic coherence value $\psi_{\rm env}$ as the correct asymptotic boundary condition, the coherence field in the galactic halo is a transition region between the inner source-pinned value and the IGM. The $1/r$ Coulomb profile of the transition, combined with the quadratic $V_{\rm eff}$ near $\psi_{\rm env}$, produces the $1/r^2$ density and flat rotation curves.

3. Source-balance in the cubic regime. Near the fold bifurcation point ψ_c, the balance between the baryonic source and the cubic term of $V_{\rm eff}$ gives $(\psi_{\rm in} - \psi_c) \propto M_{\rm b}^{-1/4}$ at fixed surface density. Matching at R_1 then yields $C^2 \propto M_{\rm b}^{1/2}$, and $v_\infty^4 \propto M_{\rm b}$.

4. Fold bifurcation as the cubic inflection. The cubic inflection at ψ_c is the fold (saddle-node) bifurcation normal form from `Coh_collapse_catastrophe` Section 4.1. The IGM stable branch minimum plays the role of $\psi_{\rm env}$; the fold point plays the role of ψ_c; the separation $\Delta = \sqrt{-2a/b}$ is set by the cosmological control parameters. No additional assumption beyond the framework's existing ingredients is required.

The anchoring acceleration is $a_0 = 9\pi G \kappa_C^2 \Sigma_0 a^2$, where a is the fold-proximity parameter measuring the IGM state's distance from the fold bifurcation surface. For cosmologically natural values of a, $a_0 \sim cH_0 \simeq 10^{-10}\,\mathrm{m\,s^{-2}}$, matching the observed MOND acceleration scale.

The framework thus predicts the Tully–Fisher relation and identifies MOND's a_0 with a cosmological-geometric quantity: the distance from the universe's coherence sector to a fold bifurcation. MOND's galactic phenomenology emerges as a consequence of galactic sources probing the near-critical structure of the cosmological coherence background, not as a modification of gravity.

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